DOCUMENT RESUME

ED 075 236 24 SE 016 065

TITLE	Self-Paced Physics, Course Materials.
INSTITUTION	Naval Academy, Annapolis, Md.; New York Inst. of Tech., Old Westbury.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO	BR-8-0446
PUB DATE	[73]
CONTRACT	N00600-68-C-0749
NOTE	236p.
EDRS PRICE	MF-$0.65 HC-$9.87
DESCRIPTORS	College Science; *Course Descriptions; *Course Organization; Curriculum Development; Educational Programs; *Independent Study; *Physics; Problem Solving; Science Education; Self Help Programs
IDENTIFIERS	Self Paced Instruction

ABSTRACT

Samples of the Self-Paced Physics Course materials are presented in this collection for dissemination purposes. Descriptions are included of course objectives, characteristics, structures, and content. As a two-semester course of study for science and engineering sophomores, most topics are on a level comparable to that of classical physics by Halliday and Resnick. Passages of four college-level physics textbooks are used as reading assignments. In the material development, emphases are placed on instructional objectives represented by core problems, an exposition through enabling and competence check problems, an iterative process of successive tryouts, and a self-instruction theory with minimum tutorial support. Contained in the whole set are 18 problems and solutions books, 72 study guides, 25 videotapes, 25 talking books, 25 illustrated texts, 12 quarterly diagnostic tests, remedial problem sets, one student manual, two instructor's manuals for course and laboratory, three laboratory manuals, and one enrichment volume. The course has been used for three years at the U. S. Naval Academy through an extensive trial-and-revision process. (Related documents are SE 016 066 - SE 016 088 and ED 062 123 - ED 062 125.) (CC)

SELF-PACED PHYSICS COURSE MATERIALS

Self-Paced
PHYSICS

CONTENTS

NEW YORK INSTITUTE of TECHNOLOGY

Self-Paced

PHYSICS

OVERVIEW

NEW YORK INSTITUTE of TECHNOLOGY

OVERVIEW

SELF-PACED PHYSICS COURSE

The *Self-Paced Physics Course* is a two-semester course in calculus-oriented, college-level physics, developed by the New York Institute of Technology for the U.S. Naval Academy with funds provided by the U.S. Office of Education. Outstanding features of the course include the imaginative use of a variety of media and materials and the extensive use of branching and self-pacing to individualize instruction.

COURSE DESCRIPTION

The *Self-Paced Physics Course* is designed to teach introductory college physics to sophomore students of science and engineering. Among the topics covered in the course are mechanics, wave phenomena, electricity, magnetism, and optics--in short, most of the topics that would be found in any introductory course in classical physics.

Each student's path through the physics course is determined by his achievement of a set of measurable behavioral objectives (MBO's) that have been designed for the course. There are over a thousand MBO's in two categories: TO's or terminal objectives, which describe the desired final student behavior, and EO's, or enabling objectives, which are steps toward the terminal behavior desired. Branching for remediation or acceleration is built into the course, so that the instruction received by any student fits his needs as precisely as possible. Further individualization is provided by the self-pacing characteristic of the course. Each student can move through the material at his own pace, going on to the next topic when he is ready. Often he can choose the medium in which he wants to study. For example, the same topic may be covered by a videotape, an illustrated text, and a "talking book" (which consists of a tape cassette and a booklet containing the diagrams referred to in the tape). The student can use the mode of instruction that is most comfortable or most successful for him.

FORMAT

The *Self-Paced Physics Course* is divided into seventy-two Segments. For each Segment there is a reading assignment in one of the standard textbooks; additional readings are assigned as options. All the practice and exercise materials are contained in a series of *Problems and Solutions* books, with three or more Segments to a book. Each Segment contains Information Panels, giving detailed information about the problems the student will encounter in that Segment. For each Segment there is a Study Guide which contains the branching steps that determine the student's path through the course material and gives detailed instructions on how to progress through the Segment. In addition, the student is frequently directed by the Study Guide to work with audiovisuals such as videotapes, talking books, or illustrated texts. Remedial problems are provided to supplement the seventy-two Segments of the standard course.

Two kinds of informal diagnostic tests are used in the course. One is called a Progress Check, and is administered after a specific number of Segments. Progress Checks are used for diagnosis, evaluation, and tutorial assistance. The other informal test is called a Periodic Diagnostic. This test form is used to diagnose possible weak areas in the student's work and to prescribe remedial work if necessary.

Formal midterm and final examinations are used to measure mastery of the course material and to determine the student's grade.

MATERIALS

The *Self-Paced Physics Course* utilizes a variety of instructional materials, including illustrated texts, standard textbooks, talking books, Study Guides, and Manuals. The Study Guides are prepared to permit the use of latent-image pens. The latent-image pen is a device designed to provide immediate feedback to students studying independently. To mark his answer, the student rubs the pen over the response box he has chosen. If his answer is right, a check mark (✓) appears in the box. If it is wrong, an "X" appears. Branching instructions are also revealed by the latent-image pen, in accordance with the student's progress. The provision of immediate feedback without the intervention of the instructor greatly increases the potential for individualizing instruction.

A list of the materials used in the course is presented below.

18 *Problems and Solutions* books, containing
Segments 1-72 of the course

72 Study Guides (latent-image printed) for
Segments 1-72

25 videotapes

25 talking books, consisting of 25 tape
cassettes and 25 booklets of diagrams

25 illustrated texts

12 quarterly diagnostic tests

remedial problems

Student Manual

Instructor's Manuals (2) for Course and Lab

3 Laboratory Manuals, containing Lab
Sessions 1-15

A volume of *Problems and Solutions* designed for enrichment of the standard course is also available.

The *Self-Paced Physics Course* has been used for three years at the U.S. Naval Academy, and has gone through an extensive trial-and-revision process.

Reprinted in its entirety from Publishers Alert Service, No. 72-03.

UNITED STATES NAVAL ACADEMY

STUDENT MANUAL

Self-Paced Physics

DEVELOPED AND PRODUCED UNDER THE
U.S. OFFICE OF EDUCATION, BUREAU OF RESEARCH,
PROJECT #8-0446, FOR THE U.S. NAVAL ACADEMY
AT ANNAPOLIS. CONTRACT #N00600-68C-0749.

NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY.

STUDENT MANUAL

1. DESCRIPTION OF THE COURSE

The self-paced physics course differs from conventional courses in a number of ways. It is largely student-managed programmed instruction. Most of your learning will be derived from reading carefully selected passages in excellent textbooks, simplified written discussions of the highlights of the various subject areas, and the use of audiovisual aids in the form of video tapes, "talking books", and brief, meaty illustrated pamphlets called Illustrated Texts. An instructor will be available for tutorial assistance as well as diagnosis of your progress.

The format of the course permits you to monitor your performance and achievement by means of instant feedback from the visual response mechanism to be described later.

In addition to self-paced theoretical instruction, you will also spend an adequate amount of time in the physics laboratory and attend a demonstration-lecture periodically.

You will always know in advance when a check quiz or an evaluation test is to be given. As a matter of fact, you will determine for yourself when progress checks will be administered to you. In addition to other periodic tests, a standard midterm and final examination will be used to evaluate your achievement.

2. COURSE STRUCTURE

Assigned reading - From standard textbooks, coded as follows:

HR means Halliday and Resnick, PHYSICS FOR STUDENTS OF SCIENCE AND ENGINEERING, fifth edition, combined form;

SZ means Sears and Zemansky, UNIVERSITY PHYSICS, third edition, complete;

AB means Albert Baez, THE NEW COLLEGE PHYSICS - A SPIRAL APPROACH, first printing;

SW means Shortley and Williams, ELEMENTS OF PHYSICS, fourth edition.

The required or prime reading assignment for each Segment of the course will be identified by one or more asterisks before the chapter numbers. The remaining reading is to be considered supplementary. A typical reading assignment and its interpretation will be presented as a sample later in this Manual.

For maximum effectiveness, all assigned reading should be completed before you begin work on the programmed instruction. This first reading need not be exhaustive because it is anticipated that you will return to certain sections of it time and time again as you work through the Segment.

Information Panels - Aside from your textbook reading, much of your factual and procedural information will come from Information Panels presented in the PROBLEMS AND SOLUTIONS booklet for each Segment. These panels are concise discussions relating to the principles and methods of solution involved in the accompanying problems. If you should find that you do not fully understand the material in the panel for a given section of your work, you would be expected to return to the textbook assignment for clarification.

Audiovisuals - These are important adjuncts to your reading and problem solving. When you are directed to work with a specified audiovisual, you will usually be given the option of selecting one of three media of presentation.

Video tape: a demonstration accompanied by a discussion that you view on the screen of a small video tape playback;

Talking Book: a set of carefully constructed pictures and diagrams accompanied by an audio tape lecture;

Illustrated Text: a set of pictures similar to those used for the Talking Book accompanied by a formal written discussion matched page by page to the illustrations.

Progress Checks - groups of relevant questions which you must answer after completing a specified number of Segments, usually three in a sequence. These checks will be used for diagnosis, progress evaluation, and tutorial assistance should the latter be needed.

Periodic Diagnostics - special test forms administered periodically to assist your instructor in diagnosing possible weak areas in your learning pattern, and to enable him to prescribe remedial work where required. The Periodic Diagnostics will also be used to evaluate your achievement.

Midterm and Final Examinations - standard examinations which provide information relative to your final grade.

Enrichment Packages - for those students whose progress warrants additional, higher level material; to be a student option.

3. PRINTED LEARNING MATERIALS

PROBLEMS AND SOLUTIONS. (Hereafter referred to as the P&S.) This is bound study material containing the work for three or more Segments in a volume. The entire course consists of 45 Segments for the semester. The P&S material in a given volume will contain blue title sheets between Segments to help you find the one you want quickly. Each P&S contains:

(a) A problem section in which the questions and numerical problems are presented in strict numerical order, to be worked on in sequence.

(b) A solution section in which the correct methods of answering questions and solving problems are presented in scrambled order. Many of these solutions are terminated by additional "true-false" questions to be answered immediately after you study the individual solutions.

(c) Information Panels strategically interspersed throughout the problem section.

STUDY GUIDE. This is just what its name implies: a written guide that you must follow step-by-step, strictly in the order presented, to work your way through the problems, information panels, audiovisuals, reading, solutions, and other check points. The remainder of this Manual will be devoted to an explanation of the way in which all these aspects of your learning are related.

4. HOW TO USE THE STUDY GUIDE

Please refer to the sample study guide which is the last page of this booklet. It is a partial mock-up of a Segment that doesn't really exist, and will be used for explanation purposes only. If you are to understand how the system works, if you are to avoid blunders when you start work on your first actual Segment, you must walk through the following explanation without missing a step. Take your time; be absolutely certain you understand each maneuver perfectly. If you need help in interpretation, ask for it.

Before you begin work on any Segment, ascertain that you have the correct STUDY GUIDE by checking the number near the upper right-hand corner, then complete the heading on each STUDY GUIDE sheet.

Another preliminary step: look at the bottom of the STUDY GUIDE sheet and note the number of pages you should have in your hand. Few STUDY GUIDES contain more than two pages. Be sure you have what you need before you start work.

The letter P above the left column means "Problem Number": the STEPS are also numbered to indicate the sequence of things you must do other than problem solving.

All right. Let's go through the sample.

Step 0.1 The reading assignment for the Segment. The required reading is in Halliday and Resnick, paragraphs 49-3 *through* 49-6 and paragraph 49-9. The slash-bar (/) always means from one paragraph through the other, inclusive. The supplementary reading is in Sears and Zemansky, paragraphs 45-6, 45-7, and 45-11. This reading should be gone through at least once before continuing.

Step 0.2 When you have finished your reading, turn to the first page in the P&S for this Segment. Read the Information Panel, be sure you understand it fully, then continue.

$\overline{\underline{1}}$ This is the first problem in the P&S. Note the overscore and underscore lines. These indicate that the problem is a core type, required of all students in the course. You will find this problem boxed for the same reason in the P&S. The problem you find in the P&S as number 1 is:

> How many gallons of regular gasoline could you have purchased with 5 Martian zilches in Septimus, Ohio in the year 1960 and still have some change left over?
>
> A. 1
> B. 2
> C. 3
> D. 4

Now obviously, to solve this problem you would have to know the price of gasoline per gallon in U.S. currency and also the equivalent buying power of a Martian zilch. Presumably, your reading and the Information Panel contains this information but let us suppose that you didn't do any of the reading and so didn't know the answer. So--you're about to make a wild guess, let's say, answer A. At this point you rub the "reveal" crayon provided all over the inside of box A for the first question. As you do so, you will see an X appear, showing that the selection was incorrect. Do it now; reveal the X in box A with your crayon. (Best results are obtained by rubbing the crayon lightly over the surface, then waiting a few moments for the revealed information to darken.)

Making another stab at it, you choose answer B and use the crayon, bringing out another X. Trying C, you find that the crayon reveals the characters 29[a]. This tells you to turn to page 29, item [a] in the P&S where you will find the full explanation of the method used to solve the problem. For this core question, you will also find a very short true-false question immediately after the correct solution. This question reads as follows:

> A Martian zilch is the equivalent of three U.S. nickels. True or False?

You must now use the reveal crayon on either the T-box or the F-box for question 1.

If you make the correct true-false selection, a ✓ will appear in the box. If you choose incorrectly, *an X will appear in the box*. The true-false questions are usually so simple that you will be permitted few, if any, errors in this part of the work. Getting one of these T-F's wrong is a pretty sure indication that you are not reading the solutions. You must avoid this.

Let's go down to the next step.

Step 1.1 You are now being given an option. If your first choice was correct, you will be permitted to skip over the next four questions and advance to the next Information Panel. If you answered incorrectly, even once, you must go through the remedial loop consisting of questions 2 through 5.

We are assuming that you missed question $\bar{\underline{1}}$, so let's go through this loop together.

2 Problem 2 in the P&S. It is not scored, hence it is not a core problem. It reads as follows:

> It is predicted that a gallon of regular gasoline will sell for $1.05 by the year 1998. If this is roughly 3-1/2 times the price of gasoline in 1960, how much did one gallon cost in 1960?

This is *not* multiple-choice. It's a completion type of question where you must write in the answer. So, write your answer on the line below the rectangle for question 2. The answer is, of course, 30¢ because $1.05 is 3-1/2 times 30¢. After writing it in, reveal the answer in the rectangle with the crayon; the answer 30¢ will appear accompanied by the referral page and item, 14[c]. Turning to the referral, you find the solution worked out for you to check your own thinking. Problems that are not core types are not accompanied by true-false check questions, so you're ready to go to question 3.

Let's interrupt the sequence for a moment. Even if you were able to answer the original core question correctly the first time, *you should go through the remedial loop anyway if you have any doubt at all about the method of solution or the answer.* You may have guessed at the right answer, or you may have made two errors that canceled out. In any case, if you feel that your choice of the right answer was a fluke in any way, we urge you to go through the remedial loop.

3 Problem 3 in the P&S; it is not a core problem. Here it is:

> Ten Martian zilches will buy exactly the same number of 2-1/2 inch Macintosh apples in a given market on a given day as two U.S. dollars. Thus, one zilch is the equivalent of
>
> A. 10¢
> B. 20¢
> C. 40¢
> D. 60¢

A glance at the STUDY GUIDE corroborates the fact that this is another multiple-choice question. Apparently 10 zilches is the equivalent of $2.00, so one zilch must be worth 20¢. This is answer B, so if you use the reveal crayon in box B you will bring out the instruction 18[b] indicating that page 18, item [b] in the P&S has the solution. Whether you were right or wrong in your selections, it is important that you read and understand the solution. If you had chosen any answer other than B, you would have revealed an X as before. There is no true-false question, hence you can now go on to question 4.

4 Here is your first modified true-false question:

> True or false? Five Martian zilches will purchase *more* milk than 20 U.S. dimes.

Note the italicized word. Read the statement and (a) if you decide it is true, simply crayon the T-box on the STUDY GUIDE; (b) if you feel that it is false, write a word that can replace *more* and thereby make the statement true. *After* you have written the correction word on the line under the F rectangle, then, and only then, you are to reveal the answer with the crayon. In this particular instance, the correct answer is "false" and you would write in the word "less" in place of *more*. Your reveal crayon will bring this out, too. If you had selected "true" as your answer, the crayon would have revealed an X inside the T-box. So, after writing "less" you would see revealed: "less (21[d])." At this point in an actual lesson, you would turn to this page and item in the P&S and read it carefully before continuing the sequence.

Continuing with the remedial loop:

5 Another multiple choice question:

> In order to have filled your 18-gallon tank with gasoline in 1960 in Septimus, Ohio, you would have spent at least
>
> A. 15 zilches
> B. 21 zilches
> C. 23 zilches
> D. 27 zilches

The correct answer is, of course, 27 zilches since each zilch is worth 20¢ and each gallon costs 30¢, so you would reveal box D and find inside the instruction "27[b]." After reading the solution, you again encounter a check T-F question which is then answered as before by revealing either the T or F box in question 5. Any answer other than D above would have revealed an X just as described for the previous multiple-choice question.

Step 5.1 Everyone is now expected to devote some time to the Information Panel, "The Currency of Venus" and then

Step 5.2 select the medium he wants for running through the audio-visual, COINAGE AND BILLS OF THE INNER PLANETS.

After that is finished, everyone starts once again on an equal footing with the core question $\underline{6}$.

And so forth.

P	STEP	NAME	P	STEP	SECTION SEGMENT 60
	0.1	Reading: HR*49-3/49-6;*49-9 SZ 45-6, 45-7; 45-11			
	0.2	Information Panel, "The Currency of Mars"			Note: In this sample "walk through", we have not included the Information Panels nor any set-up P & S. The problems that would normally appear in the P & S are given in the Student Manual for explanation purposes.
1		A B C D T F			
	1.1	If your first choice was correct, advance to 5.1; if not, continue sequence.			
2		____________ (ans)			AND SO FORTH
3		A B C D			
4		T F ____________ (ans)			
5		A B C D T F			
	5.1	Information Panel, "The Currency of Venus"			
	5.2	Audiovisual, COINAGE AND BILLS OF THE INNER PLANETS			
6		T F ____________ (ans)			
	6.1	If your answer was correct, advance to 9.1; if not, continue with sequence.			
7		____________ (ans)			

INSTRUCTOR MANUAL

UNITED STATES NAVAL ACADEMY

Self-Paced

PHYSICS

PREFACE

This manual was prepared as a reference and guide for Instructors of the Naval Academy Self-Paced Physics Course. Additional orientation is provided by the Course Manager.

Contained herein are:

1. Notes to the Instructor,
2. A description of the Management Sequence, and
3. A flow-chart which reflects a general overview of the operational functions of the course.

It is suggested that the Instructor familiarize himself with the course materials and the following student "hand-outs"-

Course Policy

The Student Manual

The Self-Paced Laboratory

NOTES TO THE INSTRUCTOR OF SELF-PACED PHYSICS

1. Introduction

The methods and operation of the self-paced physics course may seem strange to new instructors as well as to the students. This information is presented to assist the instructor in developing his individual class policies. It is presumed you are familiar with the Student Manual and Course Policy Statement.

2. Objective

The objective of the course is to enable each midshipman to complete the tasks defined by the Terminal Objectives (TOs). If you have not done so previously, you should read the TOs, as they constitute the most accurate definition of course content. Because of the way the Problem/Solution books have been constructed, successful completion of all the core questions should cover all the TOs. Since the core questions were also designed to provide a path for fast students, they are frequently complex problems that combine elements of several TOs. Due to the limited time available for testing, the body of TOs is sampled randomly during Progress Checks and Diagnostic Tests.

3. Class Atmosphere

There are few constraints on how you use class time to move the students through the material. If your class size permits, you are encouraged to use Room 203 as your regular classroom. Initially, a certain amount of encouragement may be needed to steer the midshipmen to the various media. You should try as many of the media as time permits yourself so you can recommend a particular Audiovisual if a midshipman is having trouble in a specific area. You may wish to add additional demonstrations or conduct small topical lectures occasionally. Comprehensive reviews prior to Diagnostic Tests are frequently given.

3. Student Progress

One of the by products of the course organization is the early identification of potential failures, before they reach the Diagnostic Checks. This early identification can be done most effectively by careful screening of study guide responses and progress check responses. The individual prescription for assistance is in your hands, but the early identification of these individuals and the variety of materials available should provide you with considerable flexibility.

5. Areas of Concern

a. Minimum Lecture. You, as well as some of your midshipmen, may feel uncomfortable, initially, because you are not conducting lectures during most of the class time. Experience has shown that most students adapt readily to the self-paced class routine within four to six weeks. You may choose to lecture frequently; however, you will probably have little time left to grade progress checks or counsel slow students, except in EI (Extra-Instruction) Sessions. Another by-product of the course organization is to move a substantial amount of student counseling and remedial work into the classroom.

b. Student Progress. Because of the great amount of material covered by the course, you will soon find students dropping well behind the average (or, from your view, a desirable) class progress. Your success in keeping the class moving will be limited only by your imagination. One reason for the apparently slow class progress may be confusion between a very weak physics student and a good student who chooses to "pace" himself to the speed of slower classmates. Careful screening of study guide and progress check responses can usually separate the two.

The Management Sequence

1. Each student is issued one prime textbook; at least two other supplementary texts are at all times available in physics or in the library.

2. Each student is issued a Student Manual intended to supply the student with all the procedural information required.

3. Course work begins with the issuance of Segment 1 of Problems and Solutions and the Study Guide for the same Segment. The Study Guide is a latent image type on which sequencing information is revealed by means of a special crayon.

4. The Study Guide features are:

(a) A reading assignment indicating prime reading and supplementary reading, both clearly identified.

(b) Core problems identified by score lines over and under the problem number.

(c) Remedial loop problems ("enabling problems"). The instructions for short-circuiting the loops, or following them, are contained in the Study Guide for each individual set.

(d) Titles and directions for Information Panels contained in the Problems and Solutions.

(e) Titles and directions for Audiovisuals. These are available in three formats:

(1) Video tapes;

(2) Talking Books;

(3) Illustrated Texts

(f) Homework assignment, generally in the form of additional problems in the prime text.

5. The Problems and Solutions features are:

(a) Section 1: Problems and diagrams in numerical sequence.

(b) Core problems identified by enclosing each one in a box.

(c) Information Panels preceding core groups.

(d) Scrambled problem solutions: directions for reaching solution is revealed only in the Study Guide when correct answer is chosen.

(e) Each solution for core and core-primed questions is followed by a true-false question whose answer is derivable from the solution to which it pertains. These TF's are answered in special boxed sections of the Study Guide. NOTE: Each core problem which is answered incorrectly requires that the student follow the remedial or enabling loop which always concludes with another problem having the same conceptual basis as the core problem initially missed. Such problems are called "core-primed."

(f) The scrambling process used for the solutions is extremely difficult to compromise. The time required to short-circuit the response pattern is expected to be too great to make it worthwhile.

6. The Progress Check. This is a form of test which follows a unit of work, usually three successive Segments. The Progress Check is graded by the teacher. The performance of the student is evaluated and he is then guided into one of the channels indicated below. To be eligible for the Progress Check, the student must submit to his instructor, all of the relevant revealed Study Guides for that unit.

(a) Using a predetermined cut-off grade, the student is given the "go" signal if his performance is above this level. He is also given a set of remedial suggestions in the form of reading, programmed material, films, etc.

(b) If his performance falls below the cut-off, he is given a "stop" signal with remedials, after which he re-takes a Progress Check. Questions on these checks will be randomized so that no two students ever take exactly the same examination, nor does the same student take the same check on the second round.

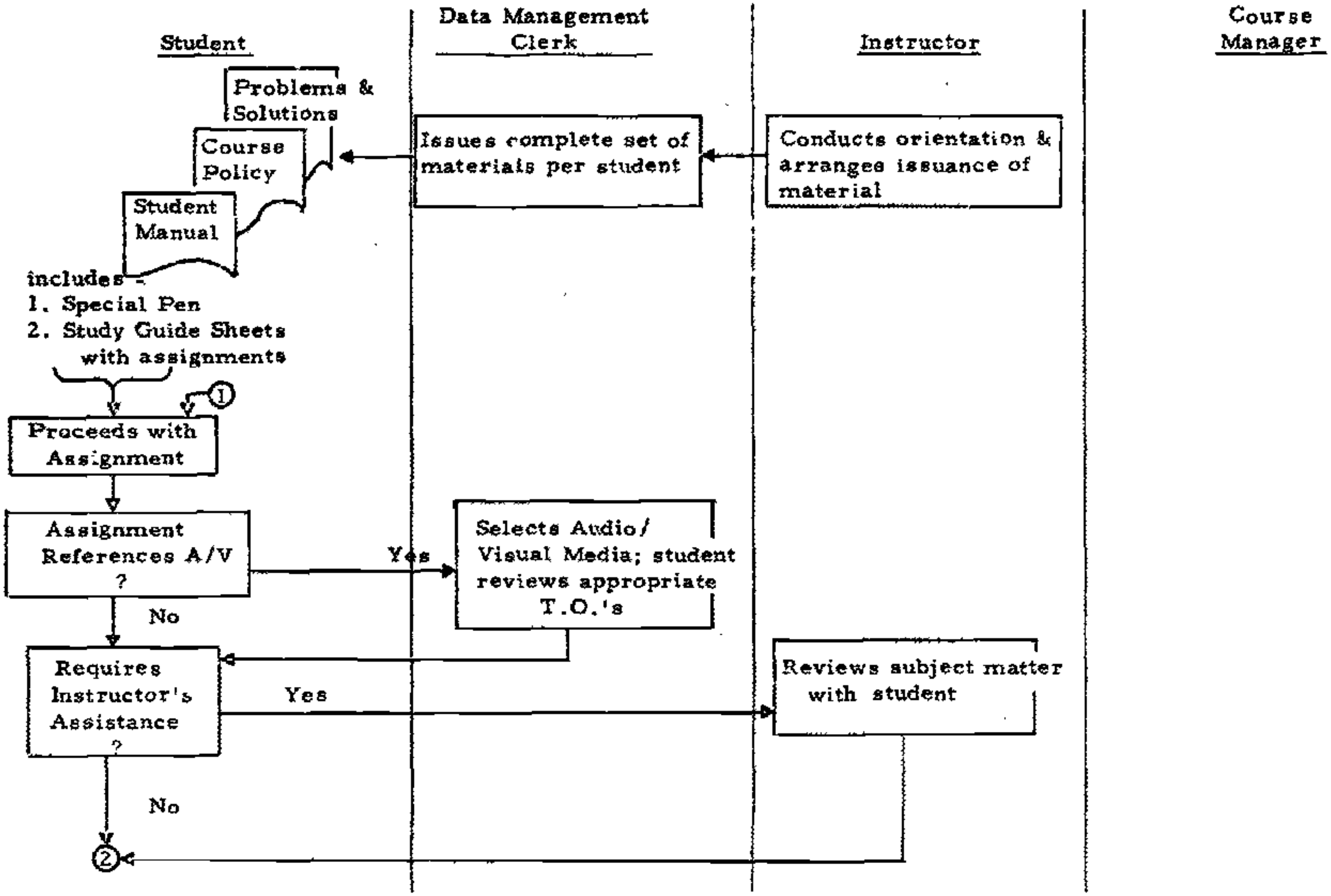
(c) If his performance falls below cut-off on the retake, he will be given individual tutorial assistance and required to take a third test. Disposition of the student after the third failure will left to the chairman of the physics committee at the Academy.

7. Quarterly Diagnostic Tests. These tests will be carefully generated to test for recognition and recall, understanding of concept, ability to recognize concepts which appear in problems, and ability to solve problems. These tests will all be of the multiple choice variety, with a response mechanism suitable for computer grading. One of the quarterly diagnostics will replace the mid-term examination and the last of them will be administered about one week before the standard final examination.

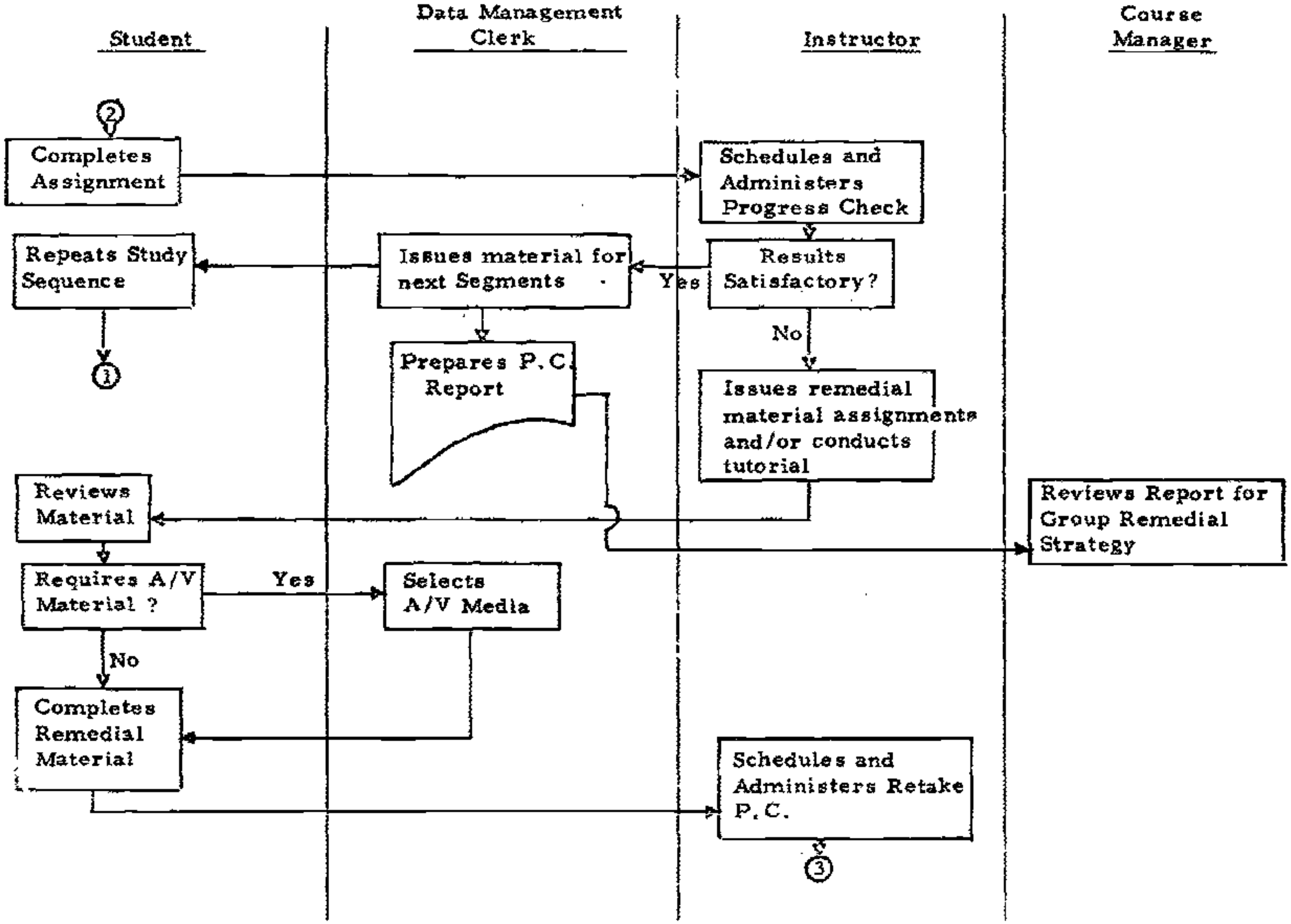
8. At the end of each quarter the instructor will submit a diagnosis and recommendations based upon study guide responses, performance on Progress Checks, and quarter diagnostics. Possible recommendations include continuation of sequence, repetition of specific segments, further use of other program texts, additional tutorials, and dropping out.

Flow Chart

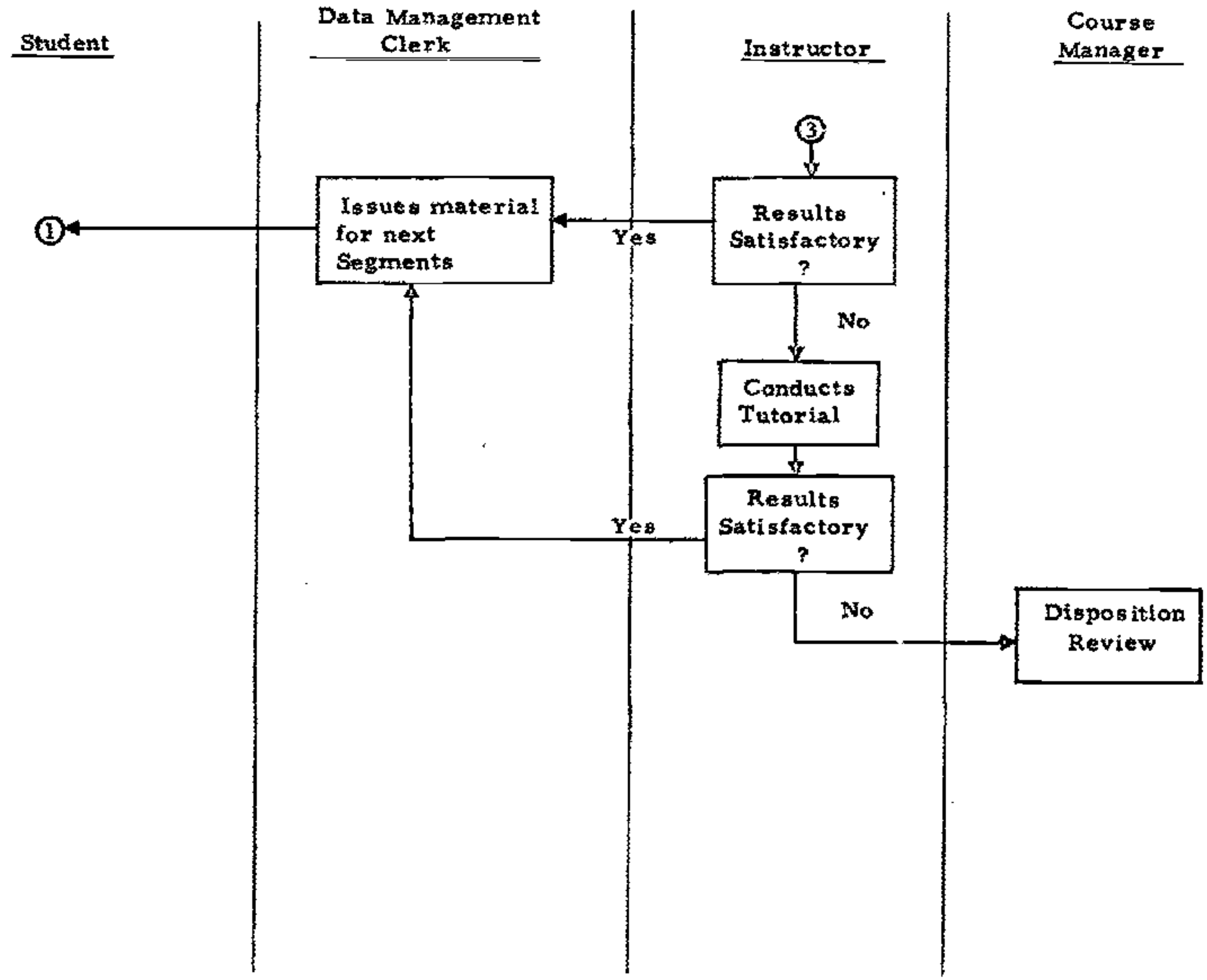
SELF-PACED PHYSICS COURSE (Page 1 of three pages)



SELF-PACED PHYSICS COURSE (Page 2 of three pages)
(Cont'd)



SELF-PACED PHYSICS COURSE (Cont'd) (Page 3 of three pages)



UNITED STATES NAVAL ACADEMY

Self-Paced

PHYSICS

SEGMENTS 6-10

DEVELOPED AND PRODUCED UNDER THE
U.S. OFFICE OF EDUCATION, BUREAU OF RESEARCH,
PROJECT #8-0446, FOR THE U.S. NAVAL ACADEMY
AT ANNAPOLIS. CONTRACT #N00600-68C-0749.

NEW YORK INSTITUTE OF TECHNOLOGY, OLD WESTBURY,

INFORMATION PANEL The Vocabulary of Circular Motion

OBJECTIVE

To define and interrelate some of the basic terms used in discussing circular motion.

As soon as you start this segment, you become involved with a few words and phrases that are worth a brief discussion.

REVOLUTIONS PER MINUTE (rev/min) and REVOLUTIONS PER SECOND (rev/sec). A particle completes a single revolution in a circle when it moves from any arbitrary starting point, around the circle, and back to the same point ready to start a second identical circular sweep. If it completes 5 such revolutions in one minute, it is said to have a FREQUENCY of 5 rev/min. If the frequency of a given particle in circular motion is 120 rev/min, it may also be expressed as 2 rev/sec.

TANGENTIAL VELOCITY (v). This is a vector quantity which expresses the instantaneous speed of the particle in a direction tangent to the circle in which it moves. If the particle's speed is constant, then the magnitude of the tangential velocity is constant but its direction is always changing. The tangential velocity vector is always perpendicular to the radius of the circle of rotation.

PERIOD (T). The period is the time required to complete one revolution. If the frequency (f) of the motion is, say, 2 rev/sec, then the period T is 1/2 sec. Clearly, period and frequency are inversely related, one being the reciprocal of the other.

$$T = 1/f \qquad \text{or} \qquad f = 1/T$$

Period may be expressed in any convenient unit of time while frequency is generally expressed in reciprocal time units. That is, the frequency of the particle above is 2 sec^{-1}.

ANGULAR VELOCITY (ω). This quantity is most conveniently expressed in radians per second (rad/sec), and is defined as the angle swept out by the radius vector of the particle per unit time. Although angular velocity is a vector quantity, we will consider only the magnitude of the vector, expressed in rad/sec, at this time. Suppose you were dealing with a particle which has a period of 1 sec. This would mean that the radius sweeps out 2π radians in 1 sec, hence the angular velocity would be 2π radians per second. If the period were half of that, that is, 1/2 sec, the motion would be twice as fast and the

next page

continued

angular velocity would then be 4π radians per second. Thus, period and angular velocity are related as follows:

$$\omega = 2\pi/T$$

and since $T = 1/f$, then:

$$\omega = 2\pi f \ .$$

DISTANCE MOVED BY PARTICLE. The instantaneous velocity of a particle in circular motion is related to the angular velocity by the expression:

$$v = \omega r$$

where r is the radius. If the particle has a constant speed of 25 cm/sec on the circle, then the distance it moves in an interval of time Δt is:

$$d = v\Delta t$$

$$= \omega r\Delta t.$$

The distance moved in 10 sec would be 250 cm. Should the angular velocity be known, the tangential speed v may be readily obtained from the expression immediately above. The radius r must, of course, be given, too.

In the portion of the work that follows, you will be asked to (1) answer descriptive questions dealing with the vocabulary presented above; (2) solve numerical problems involving these terms and phrases.

PROBLEMS

1. The rim of a rotating bicycle wheel has a tangential velocity of 30 m/sec. If 0.5 m is the radius of the rotating wheel, how many revolutions per *minute* (rev/min) would be recorded by a tachometer? (A tachometer is an instrument used to measure revolutions per minute.)

INFORMATION PANEL Characteristics of Uniform Circular Motion

OBJECTIVE

To discuss the significance of radial acceleration in uniform circular motion.

The phrase "uniform circular motion" describes a special case of circular motion in which the particle moves with constant speed, traversing equal lengths of arc in equal times. The magnitude of the tangential velocity is constant but its direction changes from instant to instant.

Students tend to think of acceleration in terms of changing speed, relating it to cars, planes, or ships as they speed up or slow down. However, the definition of acceleration includes changing direction as well as changing speed. A particle in uniform circular motion is accelerating despite the fact that the particle moves with constant speed.

The direction of the acceleration of a particle in uniform circular motion is radially inward and is called *centripetal acceleration:*

$$a_r = v^2/r \quad \text{in} \quad m/sec^2,\ ft/sec^2,\ \text{etc.}$$

where v is the tangential velocity and r is the radius of the circle. Centripetal acceleration, like linear acceleration discussed earlier, is a vector quantity.

You are expected to be able to apply the concept of centripetal acceleration to (1) answer descriptive questions in which it is involved; (2) solve problems in which centripetal acceleration must be considered.

2. A particle moves at constant speed in a circular path of radius r. The particle makes one complete revolution every second. Calculate the acceleration of the particle if r = 0.5 m.

A. 19.8 m/sec
B. 12.6 m/sec^2
C. 10.8 m/sec^2
D. 1.98 m/sec^2

3. When a particle moves in a circular path with constant speed, it is accelerating because the *magnitude* of its velocity is changing.

4. "Uniform circular motion" refers to:

A. any circular motion
B. circular motion with tangential acceleration
C. circular motion without any acceleration
D. circular motion with constant speed

5. Choose the one correct statement below pertaining to a particle in uniform circular motion.

A. Centripetal acceleration is directed radially outward from the center of the circle of motion; this acceleration arises from the change in direction of tangential velocity, but not from a change in speed.

B. Centripetal acceleration is directed radially inward toward the center of the circle of motion; this acceleration arises from the change in direction of tangential velocity, but not from a change in speed.

C. Centripetal acceleration is directed radially inward toward the center of the circle of motion; this acceleration arises from the change in direction of tangential velocity, and from a change in speed.

D. Centripetal acceleration is directed radially outward from the center of the circle of motion; this acceleration arises from the change in direction of tangential velocity, and from a change in speed.

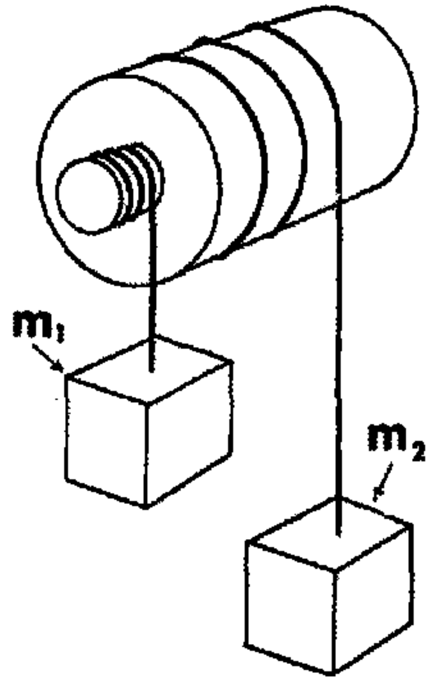
6. The magnitude of acceleration for a particle undergoing uniform circular motion is v^2/r. In this expression, r is the radius of the circle and v refers to

A. the magnitude of the tangential velocity of the particle
B. the magnitude of the outward velocity of the particle (radially outward)
C. the magnitude of the inward velocity of the particle (radially inward)
D. the radial speed in the radial direction of the particle

7. A particle of mass m is moving in a horizontal circle of radius r and making f revolutions per second. If the radius is tripled and the frequency is doubled, find the ratio of centripetal accelerations when the radius is increased from r to 3r.

A. 1/6
B. 1/12
C. 1/3
D. 2/3

8. Two blocks are lowered by a winch made of two concentric cylinders. The smaller cylinder has a radius of 0.04 m, and the larger cylinder has a radius of 1 m. If the winch turns at 3 rev/min, what are the vertical velocities of block one and block two (v_1 and v_2)?



INFORMATION PANEL Centripetal Force

OBJECTIVE

To apply the concept of centripetal force to problem situations in uniform circular motion.

You may find this line of reasoning helpful in arriving at the correct concept of centripetal force:

*If there is no unbalanced force acting on a moving particle, it will continue to move with constant speed in a straight line:

*A particle in uniform circular motion does not move in a straight line, hence it must undergo acceleration;

*This acceleration is directed radially inward, hence there must be an unbalanced force acting radially inward; we call this force the centripetal force F_c.

*The magnitude of the centripetal force is given by:

$$F_c = mv^2/r$$

When you label a force as "centripetal", you are merely stating that it acts inward toward the center of rotation, but this name gives no information about the nature of the force, nor does it tell anything about the body that is responsible for it. A centripetal force is not a new type of force but is so called only because the name is descriptive of its behavior. For a stone whirled in a horizontal circle at the end of a string, the centripetal force is an elastic force provided by the string; for a satellite revolving around the Earth, the centripetal force is gravitational attraction: a charged particle circling within the "dees" of a cyclotron is subjected to a magnetic centripetal force; and so on.

The problems you will encounter in this section are largely of the composite or multistep variety in which centripetal force is but one link in the chain of reasoning. Before starting to work out such problems, you are urged to organize your material in writing as follows:

(1) Make a list of all the "knowns" given in the problem statement in symbolic form. For example, if you are told that the mass is 2 kg, the length 98 cm, and the angle 30° (see P 8), write

<u>Given:</u> $m = 2$ kg
$L = 98$ cm $= 0.98$ m
$\theta = 30°$

(2) Write down the "unknowns", the quantity you must ultimately find, again in symbolic form. In P 8, the unknown is the period τ so

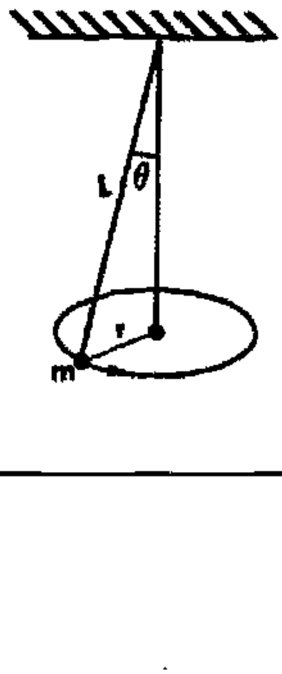
<u>Find:</u> τ (period)

(3) Next, write all the equations that appear to relate these quantities to each other and use them to help you *express all subsidiary unknowns in terms of the knowns.*

(4) You must finally obtain an equation in which only one unknown remains before you can substitute numbers.

In this section, you are expected to (1) analyze the relationships between quantities involved in describing centripetal force; (2) solve problems in which centripetal force is the central concept: (3) solve problems in which centripetal force is a subsidiary consideration, that is, only one of a number of concepts which must be interrelated to solve the problem successfully.

9. The figure shows a mass m = 2 kg revolving in a horizontal circle. The mass is suspended from a string 98 cm in length. The motion of the string traces out a cone. If the string makes an angle of 30° with the vertical, how long does it take for the mass to make one revolution?



10. A body of mass m revolves in a circle of radius r with speed v. What is the magnitude of the centripetal force, F_c, acting on the mass?

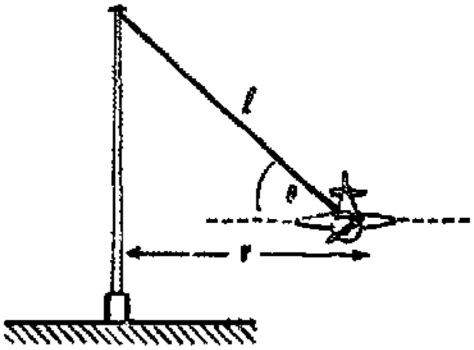
A. $F_c = mv^2/r^2$
B. $F_c = mv^2/r$
C. $F_c = mv^2r^2$
D. $F_c = mv^2r$

11. A man made satellite orbits the Earth in a circular path of radius r. The centripetal force and the force due to gravitational attraction acting on the satellite are

A. the same force
B. different forces

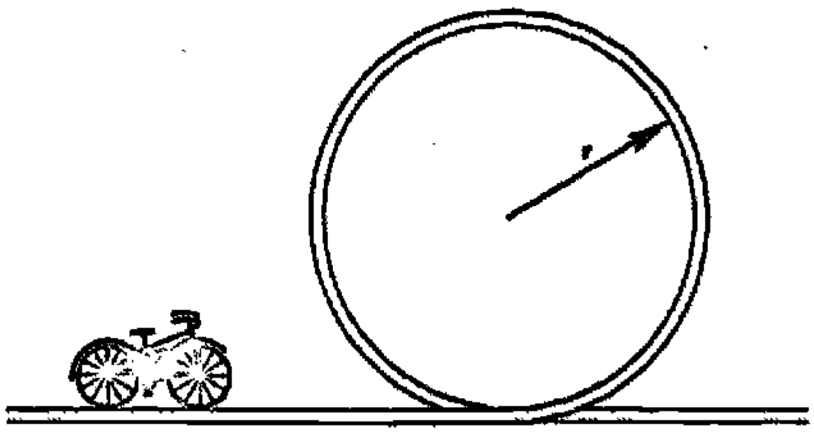
12. A boy spins a 2-kg stone in a horizontal circle at the end of a string. The string is 2 m long, and has a maximum breaking strength of 100 nt. What is the maximum speed the stone may have without breaking the string?

13. At a carnival, cars in the "airplane ride" are suspended from a cable 64 feet long attached to a vertical pole which rotates at 1 radian per second. Find the angle θ that the cable makes with the horizontal.



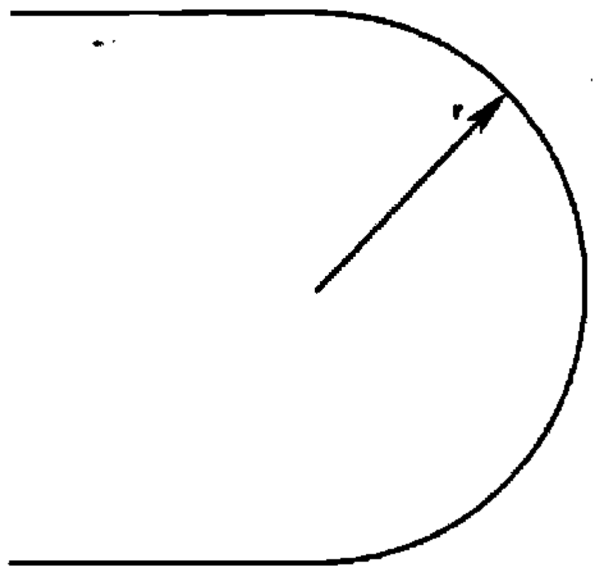
14. A copper penny is placed 4 inches from the center of a hi-fi record. The record plus penny are then placed on a phonograph turntable (33 1/3 rev/min) and the switch is turned on. The coefficient of static and kinetic friction are 0.1 and 0.05 respectively. At what angular velocity will the penny begin to slide?

15. A man plans to perform the loop-the-loop with his bicycle at the county fair (see the diagram below). The radius r is equal to 10 ft. What is the minimum speed at which he can safely perform the stunt?

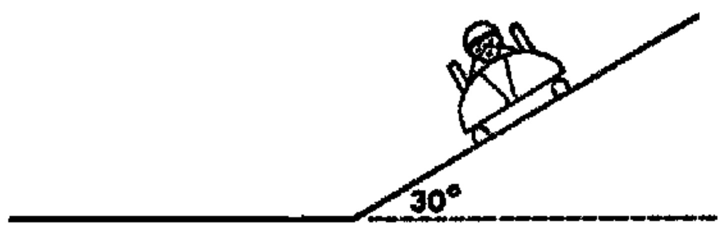


A. depends on the man's mass
B. 12.2 mi/hr
C. 20 ft/sec
D. 9.6 mi/hr

16. A bobsled speeds around the curve shown in the figure below. The curve has been well iced and can be considered frictionless. The sled moves in a circular arc of radius = 100 m and banking angle of 30°; what is its speed?



Curve view from the top



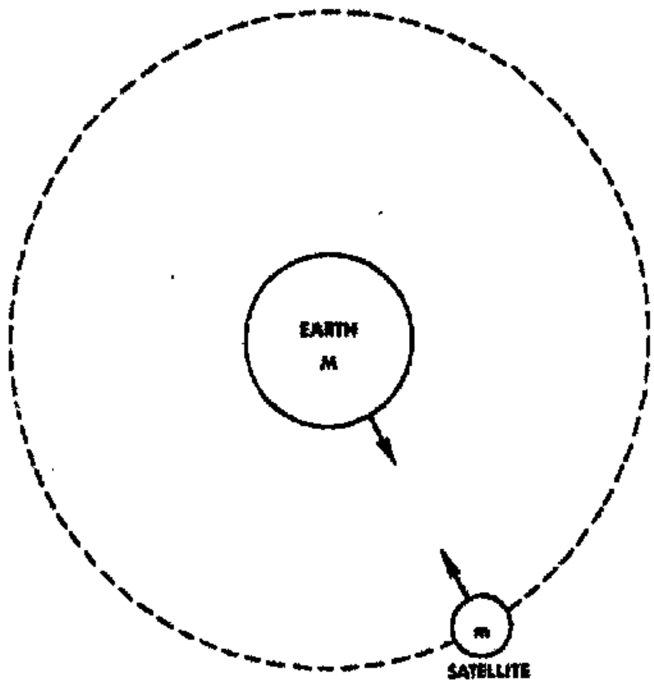
Bobsled on banked curve

[a] CORRECT ANSWER: FALSE (change *magnitude* to *direction*)

Speed has been defined as the magnitude of the velocity vector; in this problem, speed is constant. Since the particle travels in a circle, the *direction* of the velocity is changing at a constant rate. One would anticipate that the particle has a constant acceleration due to its constant change in direction.

[b] CORRECT ANSWER: A

A - As the satellite circles the Earth, only one force pulls the satellite in a circular orbit. This force is an attractive force that exists between any two masses. Since the satellite moves in a circle, this force must equal mv^2/r.



[c] CORRECT ANSWER: A

A - The tangential velocity v is related to the radial acceleration a_r by

$$a_r = v^2/r$$

[a] CORRECT ANSWER: 1.85 sec

Your reasoning may be something like this:

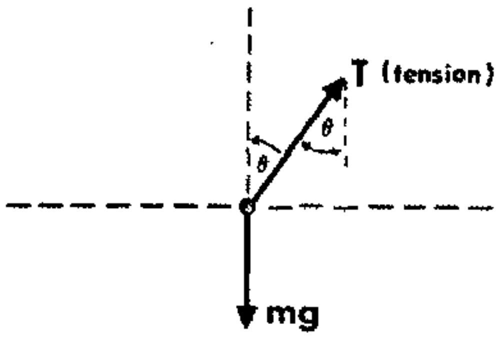
1. In order to find the time for one revolution of the mass, you must know the velocity of the mass and the path length of one revolution.

2. One revolution is $2\pi r$ in length where r is the radius of the horizontal circle in which the mass moves.

3. The magnitude of the tangential velocity v can be found from its relationship to the centripetal force, F_c,

$$F_c = mv^2/r$$

4. In order to determine this force, a free-body diagram is in order.



5. The only force acting in the radial direction is $T \sin 30°$ and thus this force must be equal to the centripetal force.

Now in equation form: $\Sigma F_y = T \cos 30° - mg = 0$ (because $a_y = 0$)

$T \sin 30° = mv^2/r$ (because the mass moves in a circle)

Solving for v we obtain

$$v = \sqrt{rg \tan 30°}$$

and, of course, from the diagram, $r = L \sin 30°$.

Finally, the time for one revolution would be

$$\tau = \frac{\text{distance}}{\text{velocity}} = \frac{2\pi r}{\sqrt{rg \tan 30°}} = \frac{2\pi\sqrt{r}}{\sqrt{g \tan 30°}} = 2\pi\sqrt{\frac{L}{g}\cos 30°} = 1.85 \text{ sec}$$

TRUE OR FALSE? The angle θ in the free-body diagram is 30° for this problem.

[a] CORRECT ANSWER: 573 rev/min

The distance traveled in one revolution is equal to the circumference, or $2\pi r$, of the circle. This distance divided by the velocity would give us the time for one revolution. The time for one revolution is called the period and is given by the symbol T.

$$T = \frac{2\pi r}{v} = \text{time per revolution}$$

You might also notice that

$$\frac{1}{T} = \text{revolutions/time} = \text{frequency} \equiv f$$

which is the required quantity.

We have

$$f = v/2\pi r$$
$$= 9.55 \text{ revolutions/sec}$$

This can be converted to revolutions per minute as follows:

$$f = 9.55 \frac{\text{rev}}{\text{sec}} \times 60 \frac{\text{sec}}{\text{min}} = 573 \frac{\text{rev}}{\text{min}}$$

TRUE OR FALSE? The period of a rotation increases as the number of revolutions per minute increases.

[b] CORRECT ANSWER: B

B - The word "centripetal" means center-seeking or toward the center. The velocity of a particle in uniform circular motion changes in direction, but the speed (or magnitude) remains constant.

[c] CORRECT ANSWER: D

D - The case of a particle moving in a circle with constant speed is called uniform circular motion. The velocity vector changes continuously in direction but not in magnitude.

[a] CORRECT ANSWER: B

B - The most dangerous part of the stunt will come when the man on the bicycle is upside down at the top of the loop. At that point he is most likely to leave the track. "Just about to leave the track" means that the normal force due to the surface of the track is approaching zero. Therefore, we should solve for this extreme condition to obtain our minimum safe speed.

At the top point, traveling at a velocity v, the only force acting on the man is his weight. Therefore, we obtain

$$\Sigma F_y = -mg = ma$$

This tells us the acceleration is downward, in the radial direction, and equal to the acceleration of gravity. Then

$$a = a_r = v^2/r$$

and

$$v = \sqrt{rg}$$

TRUE OR FALSE? The minimum permissible speed of the bicycle in a loop-the-loop situation is independent of the mass of bicycle and rider.

[b] CORRECT ANSWER: B

B - For a particle moving in a circle of radius r and frequency f, the centripetal acceleration is given by

$$a_r = \frac{v^2}{r} = \omega^2 r = 4\pi^2 f^2 r$$

Let a_{1r} be the acceleration when the radius is r and a_{3r} be the acceleration when the radius is 3r. Therefore,

$$\frac{a_{1r}}{a_{3r}} = \frac{4\pi^2 f^2 r}{4\pi^2 (2f)^2 (3r)} = \frac{1}{12}$$

TRUE OR FALSE? In this problem, a_r symbolizes tangential acceleration.

[a] CORRECT ANSWER: 10 m/sec

Some basic facts about circular motion:

1. Circular motion is always accelerated motion. This is due to the fact that even if the speed is constant, there is a constantly changing direction.

2. From Newton's second law, we know that acceleration implies the presence of a force. In a uniform circular motion, this force is called centripetal force. If the centripetal force were not present, the object would fly off at a tangent to the circle. Thus, the centripetal force is the force that holds the object in a circular orbit. The magnitude of the forces can be found from the following:

$$F = mv^2/r$$

where

v = tangential velocity (speed)

r = radius, and

m = mass

Most problems involving uniform circular motion require that the total forces which constrain a body to move in a circle are equated to mv^2/r.

In the present problem, when $F = 100$ nt and $r = 2$ m, the speed of the stone is

$$v = \sqrt{Fr/m}$$

$$= 10 \text{ m/sec}$$

[a] CORRECT ANSWER: 30 rev/min

The only force available for keeping the penny in a circular path is the force of friction (static). Let's solve for the angular speed at which the static frictional force would be maximum.

$$\Sigma F_y = N - mg = 0$$

$$\Sigma F_x = \mu_s N = ma_r$$

where μ_s is the coefficient of static friction and

$$a_r = \text{radial acceleration} = \omega^2 r$$

Therefore,

$$\mu_s mg = m\omega^2 r$$

Solving for ω, we obtain

$$\omega = \sqrt{\frac{g\mu_s}{r}}$$

Substitution of numerical values yields

$$\omega = \sqrt{\frac{32 \times 0.1}{1/3}}$$

$$\omega = 3.1 \text{ rad/sec}$$

Therefore,

$$\text{rev/min} = 3.1 \frac{\text{rad}}{\text{sec}} \times 60 \frac{\text{sec}}{\text{min}} \times \frac{1}{2\pi} \frac{\text{rev}}{\text{rad}} = 30 \text{ rev/min}$$

Thus the penny will slide off when the turntable attains the speed of 30 rev/min.

TRUE OR FALSE? The coefficient of kinetic friction must be used in the solution of this problem.

[a] CORRECT ANSWER: B

B - One way to check the answer is to check the dimensions of the answer. Observe that the dimensions of your answer agree with the dimensions ML/T^2 of any force, where M = mass, L = length, and T = time. In our problem,

$$F = mv^2/r,$$

which has dimensions

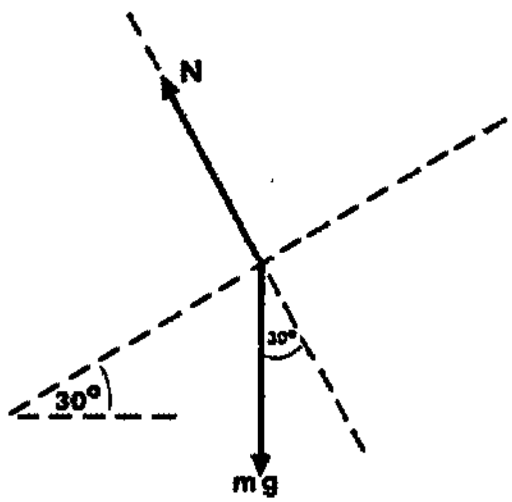
$$ML^2/T^2L$$

reducing to the dimensions of force, ML/T^2.

Remember that this force is always directed towards the center of the circle.

[b] CORRECT ANSWER: 23.8 m/sec

A free-body diagram of the sled in the curve would indicate only two



forces acting on the sled. The normal force must provide a component to balance the weight and a component to cause the radial acceleration,

next page

continued

$\Sigma F_y = N\cos 30° - mg = 0$ (from $a_y = 0$)

$N \sin 30° = mv^2/r$ (radial force must equal mv^2/r to maintain circular motion)

Eliminate N and solve for v:

$$v = \sqrt{rg \tan 30°}$$

Notice that the mass of sled and passengers is not important, so the bob-sled course is properly banked for people of all sizes.

TRUE OR FALSE? The vertical component of the normal force N is not equal in magnitude to the weight mg.

[a] CORRECT ANSWER: C

C - In all problems one should carefully decide on three distinct questions:

(a) What must I find? "Unknowns"

(b) What information has been given? "Knowns"

(c) What equations do I know that relate the "knowns" to the "unkowns"?

In this problem for example,

GIVEN	FIND
1) constant speed	radial acceleration (a_r)
2) circular path	
3) radius of circle	
4) time to travel one revolution	

Since the speed is constant, the particle has only radial acceleration and no tangential component of acceleration. An equation relating some of these quantities is

$$a_r = v^2/r$$

next page

continued

We must determine the velocity, v.

Possible Equation: $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$

We know that the particle travels one revolution in one second. The distance traveled in one revolution is the circumference of the circle, or $2\pi r$. Therefore,

$$v = \frac{2\pi r}{1} \frac{\text{meters}}{\text{second}}$$

Hence $$a_r = \frac{4\pi^2 r^2}{r} = 4\pi^2 r$$

TRUE OR FALSE? The distance traveled by a particle in making two complete revolutions in circular motion is $4\pi r$ where r is the radius of rotation.

(a) CORRECT ANSWER: $v_1 = 0.01256$ m/sec, $v_2 = 0.314$ m/sec

The ropes unwind at the same speed with which the cylinders turn; that is, their tangential velocities. Tangential velocity v is directly proportional to the radius r. As you might recall,

$$v = \omega r$$

where ω = angular velocity in radians/second.

In our problem,

$$\omega = 3 \frac{\text{rev}}{\text{min}} \times \frac{1}{60} \frac{\text{min}}{\text{sec}} \times 2\pi \frac{\text{rad}}{\text{rev}} = 0.314 \frac{\text{rad}}{\text{sec}}$$

and $$v_1 = \omega r_1$$

becomes $$v_1 = 0.314 \frac{\text{rad}}{\text{sec}} \times 0.04 \text{ m} = 0.01256 \text{ m/sec}$$

Similarly, $$v_2 = \omega r_2$$

and gives $$v_2 = 0.314 \frac{\text{rad}}{\text{sec}} \times 1.0 \text{ m} = 0.314 \text{ m/sec}$$

TRUE OR FALSE? In this problem, the angular velocity of the smaller cylinder is exactly the same as the angular velocity of the larger cylinder.

[a] CORRECT ANSWER: 30°

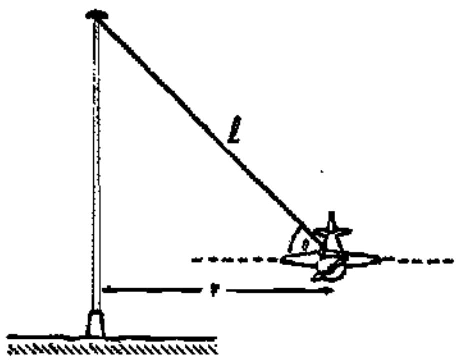
Using Newton's second law, the sum of the horizontal forces, ΣF_h, is

$$\Sigma F_h = T\cos\theta = ma \qquad (1)$$

Since the particle is moving in a circular path with constant speed, the acceleration a is the centripetal acceleration. Therefore, equation (1) may be written as

$$T\cos\theta = m\omega^2 r$$

where r is the radius of the circular path.



From the diagram

$$r = \ell\cos\theta$$

Substitution of $r = \ell\cos\theta$ in equation (1) yields

$$T\cos\theta = m4\pi^2 f^2 \ell\cos\theta \qquad (2)$$

The sum of the forces in the vertical direction, ΣF_v, is

$$\Sigma F_v = T\sin\theta - mg = 0$$

or

$$T\sin\theta = mg \qquad (3)$$

Dividing equation (3) by equation (2) we obtain

$$\sin\theta = \frac{g}{\omega^2 \ell}$$

$$= 1/2$$

Therefore, $\theta = 30°$

TRUE OR FALSE? The centripetal force acting on the carnival car is the horizontal component of the tension in the string.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Work Done by a Constant Force

OBJECTIVE

To calculate the work done by a constant force, that is, a force which varies neither in magnitude nor direction.

In the simplest situation, where the force applied to a body is constant in both direction and magnitude and where the resulting motion occurs in a straight line, we define work as the product of the magnitude of the force and the displacement of the particle on which the force acts.

Since force and displacement are both vectors, care must be taken to use a consistent system of symbols. In our work we will continue to use $\vec{r}$ for the position vector. Displacement will be designated by $\vec{s}$ so that a particle moving from position $\vec{r}_1$ to $\vec{r}_2$ will undergo a displacement of $\Delta\vec{r} \equiv \vec{s} = \vec{r}_2 - \vec{r}_1$. Thus, with this convention, $d\vec{s} \equiv d\vec{r}$ and the two differentials may be used interchangeably although $d\vec{s}$ will be the preferred form.

The work W done by a constant force $\vec{F}$ acting on a body which moves through a displacement $\vec{s}$ is $W = \vec{F}\cdot\vec{s} = Fs\cos\theta$ in which θ is the angle between the two vectors.

If we designate the component of the force in the s-direction as F_s, then

$$F_s = F\cos\theta$$

and so
$$W = F_s s$$

In working through the problems dealing with the work done by a constant force, you will be expected to

(a) justify the conclusion that the work done by centripetal force on a particle moving uniformly in a circle is zero;

(b) calculate the work done on a given mass when moved up an incline by a given distance;

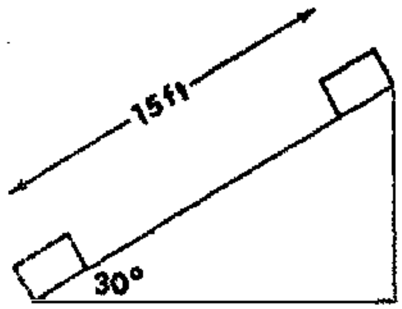
(c) find the work done on a given mass when lifted vertically over a given distance.

PROBLEMS

1. A 2-kg particle is moving in a circle with an angular velocity of 10 rad/sec. The diameter of the circle is 1 m. How much work is done on the particle by the centripetal force during one revolution?

A. 400π J
B. 200π J
C. 100π J
D. Zero J

2.

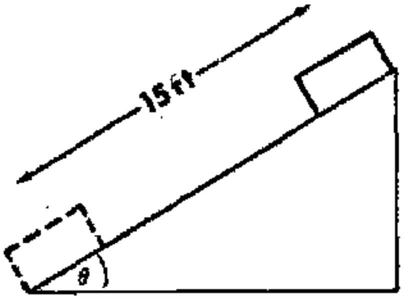


A safe having a mass of 2 slugs is moved up a 30° frictionless inclined plane for a distance of 15 ft. Calculate the work done on the safe.

3. A book of mass m is lifted a vertical distance y near the surface of the Earth. Which of the following expressions gives the work done on the book?

A. (1/2)my
B. my
C. (1/2)mgy
D. mgy

4. A 3-kg block is pushed 15 ft along a frictionless inclined plane. The work expended is 720 ft-lb. Find the angle of incline θ.



INFORMATION PANEL — Work Done by a Varying Force

OBJECTIVE

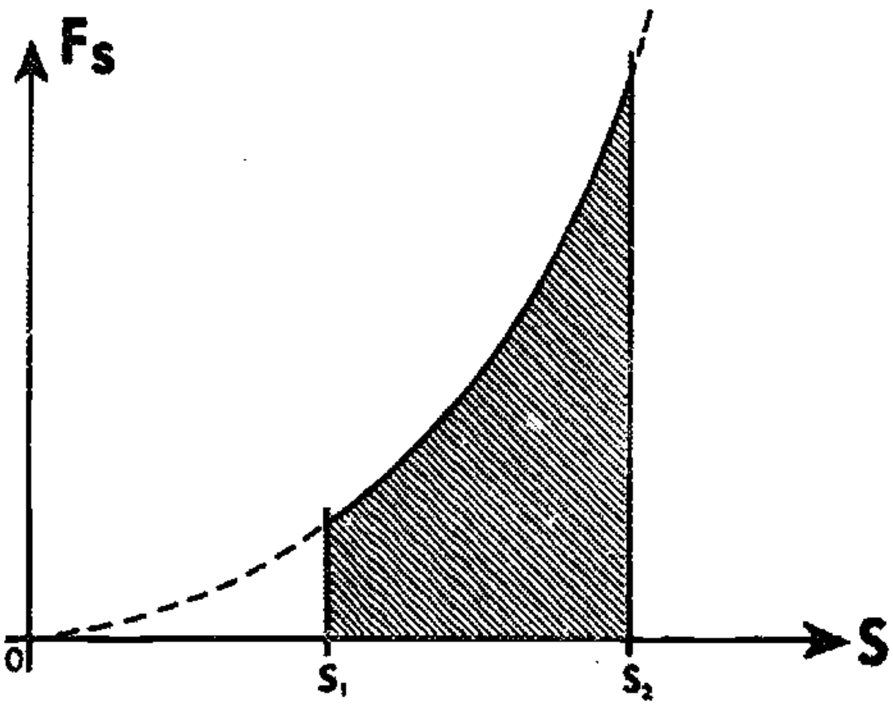
To calculate the work done by a force that varies in a known manner.

The force that causes the displacement of a particle often changes in magnitude and/or direction from instant to instant in a mathematically predictable way. For example, the magnitude of a force is often a function of the displacement of the particle to which the force is applied: as a spring is stretched or compressed, the force needed to produce each successive increment of displacement becomes larger as the end of the spring moves farther from its rest position. In this case, the work done is no longer a simple product of force and displacement.

When the force varies in magnitude, or when the force and displacement are not in the same direction, the work done is given by

$$W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s} = \int_{s_1}^{s_2} F_s ds$$

In the graph of F_s vs s, it can be seen that the work due to a displacement from s_1 to s_2 is the area under the curve between the end points s_1 and s_2.



You will find the problems in this section require that you be able to

(a) calculate work when the force is given as a function of displacement (in equation form);

(b) calculate work when the dependence of force on displacement is shown in the form of a graph;

(c) calculate work in the special cases when the force is a simple restoring force, or a gravitational force near the Earth's surface.

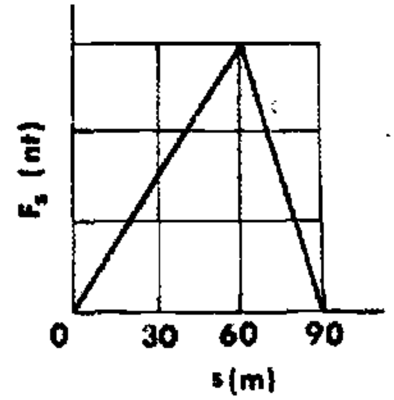
5. A mass $m = 2$ kg moves in the direction of an applied force varying with displacement according to the equation

$$F = m(\alpha + \beta x^2)$$

where $\alpha = 5\ m/sec^2$, $\beta = 15\ m^{-1}sec^{-2}$, and x is the displacement. Find the work done on the mass during the first 2 m of its journey.

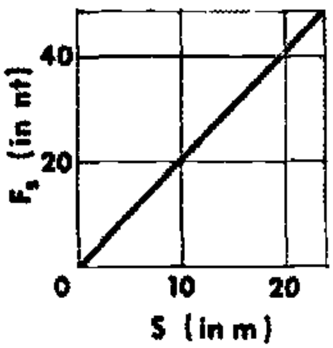
A. 260 j
B. 130 j
C. 100 j
D. 20 j

6.



The graph shows the dependence of F_s on the displacement s. It is found that the work done from $s_1 = 0$ m to $s_2 = 90$ m is equal to 450 j. Find the maximum value of F_s.

7.



The diagram shows the linear dependence of F_s on the displacement s. Compute the amount of work done between $s_1 = 10$ m and $s_2 = 20$ m.

8. A mass m = 2 kg moves in the direction of an applied force varying with displacement x according to the equation

$$F = m(ax + b)^2$$

where $a = 1.0\ m^{-1/2}sec^{-1}$, and $b = 2.0\ m^{1/2}sec^{-1}$. Find the work done on the mass during the first 3.0 m of its journey.

A. 130 j
B. 110 j
C. 94 j
D. 78 j

INFORMATION PANEL — Algebraic Signs in Work Problems

OBJECTIVE

To recognize that work may be designated as either positive or negative and to use the accepted sign convention in solving problems that call for it.

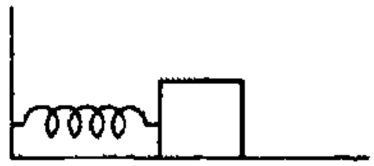
It is convenient to assign an algebraic sign to work under certain conditions. Fundamentally, the sign indicates which "body" actually does the work.

Suppose we are trying to find the work done by a body A *on* body B and it happens that body B is actually doing the work on body A. Then as far as our original assumption is concerned, that is, that body A is doing the work on body B, we would have to say that the work W_{AB} is a negative quantity.

By convention, the work done by a physical system on its environment is taken as positive. If the work comes out negative, then we know that the environment had done work on the system rather than the other way around. In problem 9 that follows, the system consists of a spring resting on a frictionless table with one end of the spring fixed in position; the "environment" in this problem is a 5-kg mass. As you work through the problem, the need for the sign convention will become apparent.

next page

continued



A spring is said to obey Hooke's law if the force necessary to stretch or compress the spring is directly proportional to the amount of stretching or compression. Within limits, most springs obey Hooke's law.

If the equilibrium position of the free end of a spring is x_o, the force $\vec{F}'$ necessary to stretch the spring to a new position x is given by

$$\vec{F}' = k(x - x_o)$$

where k is the force constant of the spring. In accord with our convention, the work done by the spring on its environment is positive.

From Newton's third law, the force of the spring on the block is

$$\vec{F} = -\vec{F}' = -k(x - x_o)$$

or, written in scalar form,

$$F = -k(x - x_o)$$

In working through the following problems, it is anticipated that you will be capable of

(a) recognizing that the magnitude of the force at any instant depends on the displacement of the body;

(b) computing the work done on the system or by the system, depending on the algebraic sign of the final answer.

9. A 5-kg block is attached to the spring shown in the diagram at the top of the page. The spring, when unstretched, has a length of 0.15 m (including the block), and its force constant k is equal to 2000 nt/m. Compute the work done in moving the block from x_1 = 0.10 m to x_2 = 0.25 m.

A. -750 j
B. 52.5 j
C. -7.5 j
D. -52.5 j

INFORMATION PANEL — Unit Systems for Work and Power

OBJECTIVE

To solve problems in work and power using consistent units throughout the solutions.

For your convenience, we have listed the power and work units commonly used in the three systems with which we deal in this course.

	WORK (energy)	POWER
MKS	joule (j) or newton-meter (nt-m)	watt (w) or joule per second (j/sec)
British	foot-pound (ft-lb)	foot-pound per second (ft-lb/sec) foot-pound per minute (ft-lb/min)
CGS	erg or dyne-centimeter	erg per second (erg/sec)
Misc.	British thermal unit (Btu) kilowatt-hour (kw-hr)	horsepower (hp)

Useful Conversions

1 watt-sec = 1 joule = 1 nt-m
1 erg = 1 dyne-cm = 10^{-7} j
1 hp = 550 ft-lb/sec = 33,000 ft-lb/min = 746 w
1 kw-hr = 3.6×10^6 j
1 Btu = 778 ft-lb

The questions and problems in this section require that you be able to

(a) use the relationship $P = \vec{F} \cdot \vec{v}$ in numerical and symbolic work;

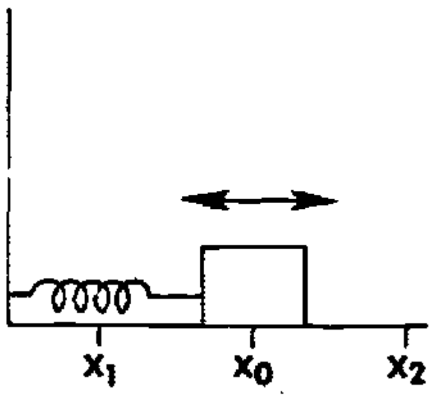
(b) find the instantaneous power supplied by an oscillating spring at a given displacement;

(c) combine work, power and friction concepts in numerical applications.

10. An escalator, inclined at 37° from the horizontal, has a motor that can deliver a maximum power of 10 hp. If the escalator is moving with a constant speed of 2 ft/sec, what is the maximum number of passengers, with an average weight of 150 lb, that the escalator can handle? (Neglect frictional losses.)

A. 30
B. 18
C. 41
D. 31

11.



A mass m attached to a spring with force constant k is oscillating back and forth between points x_1 and x_2 on a frictionless horizontal table. What is the instantaneous power supplied by the spring at the instant the mass passes through the midpoint x_0? (In the following, v is the speed of the mass at the moment it passes through point x_0.)

A. kx_0v
B. $k(x_2 - x_0)v$
C. $k(x_2 - x_1)v$
D. zero

12. A constant horizontal force of magnitude F is required in order to slide a 100-kg block on a horizontal floor with a constant speed of 5 m/sec. The coefficient of kinetic friction between the block and the floor is 0.2. How much power must be supplied by the agency responsible for the force F?

13. A 2500-lb car maintains a constant speed of 60 mi/hr up a road inclined at 20° from the horizontal. Assume that frictional forces can be neglected. How much power does the engine of the car develop?

14. An escalator, inclined at 37° from the horizontal, has a motor that can deliver a maximum power of 5 hp. If the escalator is moving with a constant speed of 1 ft/sec, what is the maximum number of passengers, with an average weight of 150 lb, that the escalator can handle? (Neglect frictional losses.)

A. 18
B. 30
C. 31
D. 41

<u>INFORMATION PANEL</u> <u>Kinetic Energy</u>

OBJECTIVE

To apply the definition of kinetic energy to descriptive questions; to solve numerical problems involving kinetic energy only.

The kinetic energy of a moving mass is defined as its ability to do work by virtue of its motion. Energy of any kind is a scalar quantity; kinetic energy K is determined for any given body by the relationship

$$K = 1/2\ mv^2$$

in which m = the mass of the body and v = the magnitude of its velocity, or its speed.

next page

continued

A mass moving with constant speed has a constant kinetic energy. The kinetic energy of a mass moving with changing speed varies from instant to instant, hence the instantaneous kinetic energy is a function of the instantaneous speed at the moment in question.

As a check on your understanding of kinetic energy, you must be able to

(a) solve a problem involving the kinetic energy of a projectile at the highest point of its trajectory;

(b) determine the kinetic energy of a given moving mass, giving attention to units, their conversion, and their consistency.

15. A 2-kg particle is projected from ground level with an initial velocity of 20 m/sec, at 60° above the horizontal. Find the kinetic energy of the particle when it reaches its highest altitude; i.e., where the vertical component of the velocity is zero. (Neglect air resistance.)

16. A 3000-lb car is moving with a speed of 60 mi/hr. Find its kinetic energy.

17. The kinetic energy of a 2-kg projectile at the highest point in its trajectory is 25 joules. It is known that the projectile was fired at an angle of 60° from the horizontal. Find the initial speed of the projectile.

OBJECTIVE

To define the work-energy theorem; to use the theorem for solving symbolic and numerical problems.

The work-energy theorem for a particle states that the work done by the *resultant* force applied to a particle is equal to the change in the kinetic energy of the particle. Or

$$W_R = K - K_o = \Delta K$$

wherein W_R = the work due to the resultant force; K = the kinetic energy of the particle after the work has been done; K_o = the initial kinetic energy before application of the resultant force.

The following can be deduced from the work-energy theorem:

1. For a particle moving with constant speed, there is no change of kinetic energy, hence the work done by the resultant force is zero.

2. The speed of the particle along a given line of motion can be changed only when the resultant force has a component along that line.

3. If the kinetic energy of a particle diminishes, the work done by the resultant force is *negative*; if the kinetic energy increases, the work is *positive*.

4. The kinetic energy of a mass in motion equals the work it can do before it is brought to rest.

5. The units used for work and kinetic energy are identical.

The following problems call for the ability to

(a) solve a problem in kinematics by means of the work-energy theorem;

(b) solve a problem involving a projectile fired at an angle to the horizontal in which you are to determine its kinetic energy upon returning to earth;

(c) determine the work required to double the speed of a given particle;

(d) use the work-energy theorem in calculating the force acting on a body, given the factors needed to find its kinetic energy before and after the force has acted.

18. A block is projected with an initial speed of 8 m/sec, down a frictionless plane inclined 45° from the horizontal. Find the speed of the block after it has traveled for a distance of 2.6 m along the incline. (Use the work-energy theorem in your solution.)

19. A constant force $\vec{F}$ is used to accelerate a body of mass m to a velocity $\vec{v}$. The force is then removed. How much work is done on the object?

A. Fv
B. $mv^2/2$
C. $(F^2/2m)t^2$
D. Fvt

20. A particle of mass m is moving with a speed v. How much work must be done on the particle in order to double its speed?

A. $(1/2)mv^2$
B. mv^2
C. $(3/2)mv^2$
D. $2mv^2$

21. A 2-kg sphere is projected with an initial velocity of 10 m/sec, at 39.3° above the horizontal, from the ground level of a horizontal field. What is the sphere's kinetic energy at the instant it hits the ground? (Neglect air resistance.)

A. 296 j
B. 256 j
C. 196 j
D. 100 j

22. A bullet having a weight of 1 oz, moving with a speed of 600 mi/hr, penetrates a tree trunk to a depth of 11 inches before coming to rest. Calculate the average force exerted on the bullet.

23. A block is projected upward on a frictionless inclined plane with an initial speed of 10 m/sec. The plane is inclined at 45° from the horizontal. Find the speed of the block after it has traveled a distance of 2.6 m along the incline.

INFORMATION PANEL Composite Problems Involving Work and Energy

---

OBJECTIVE

To combine the concepts of work and energy with other kinematic and dynamic aspects of physics in solving composite problems.

You have been made aware of the necessity for organizing your work before you begin to substitute numbers in equations dealing with composite problems. All the rules previously described--itemization of the knowns and unknowns, and writing down the interrelating equations--apply equally well to this section of your work.

The remaining problems in this segment are such composite problems. In working these out, you are expected to use the work-energy theorem in solving problems which require that you

(a) first use Newton's second law to calculate an unknown force and then apply the theorem;

(b) make use of the concepts of friction on an inclined plane to find the speed of a block on the plane after it has traveled a specified distance under the action of a constant force;

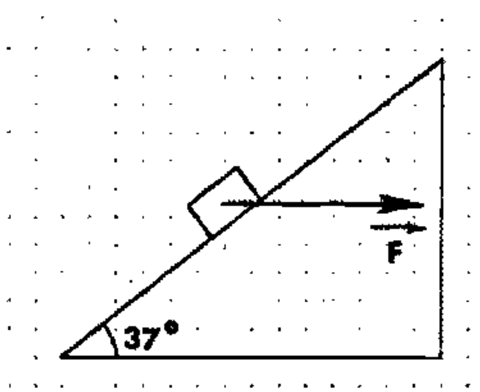
(c) calculate the speed acquired by a mass as the result of the decompression of a spring.

24. A 30-gm bullet, fired with a speed of 300 m/sec, passes through a telephone pole 30 cm in diameter at a point 2 m above ground. The bullet's path through the pole is horizontal and along a diameter. While in the pole the bullet experiences an average force of 2500 nt. If air resistance is neglected, at what horizontal distance from the pole will the bullet hit the ground?

25. A block weighing 16 lb is initially at rest. It is made to move through a distance of 100 ft in 10 sec by a constant force. Find how much work must have been done.

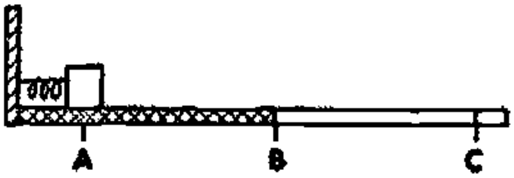
26. A 100-gm rubber ball is held at the bottom of a barrel full of water and then released. While rising to the surface, the ball experiences an average upward force of 490 dynes (this *includes* the weight of the ball). The barrel is 1000. cm high. Find the maximum distance through which the ball will rise above the barrel.

27. A constant horizontal force $\vec{F}$, of magnitude 120 nt, is used to move a 10-kg block up a plane inclined at 37° from the horizontal. If the block starts from rest, and the coefficient of kinetic friction between the block and the plane is 0.200, what is the speed of the block after it has traveled 10 m along the plane?



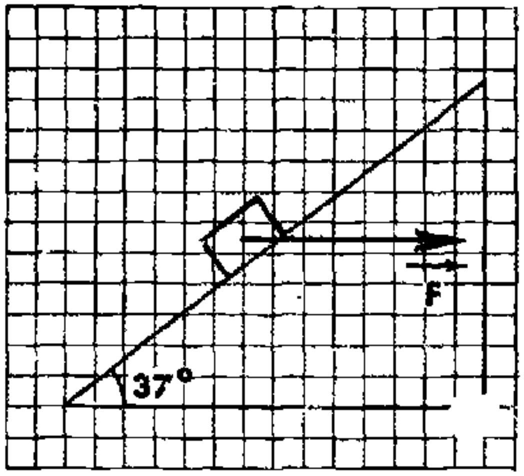
A. 6.56 m/sec
B. 9.55 m/sec
C. 12.8 m/sec
D. 3.76 m/sec

28. A 2-kg block is held at point A on a horizontal floor, with a compressed spring placed between the block and a wall but not attached to the block. The floor surface to the right of B is frictionless. The coefficient of kinetic friction between the surface to the left of B and the block is 0.25. The spring has a force constant of 573 nt/m, and it is compressed to a length 20 cm shorter than its normal length. The distance from A to C is 1 m, and B is midway between A and C. If the block is suddenly released, what will be its speed when it goes past point C?



A. 2.56 m/sec
B. 3.00 m/sec
C. 5.12 m/sec
D. 3.16 m/sec

29. A constant horizontal force $\vec{F}$, of magnitude 120 nt, is used to move a 10-kg block up a plane inclined at 37° from the horizontal. If the block starts from rest, and the coefficient of kinetic friction between the block and the plane is 0.200, how far has the block traveled along the plane when its speed is 2.1 m/sec?



A. 3.1 m
B. 4.1 m
C. 5.1 m
D. 6.1 m

[a] CORRECT ANSWER: 10 nt

The work done is equal to the area of the triangle in the figure. The area of a triangle is given by (1/2)(base) × (height).

Thus, $$W = \frac{1}{2}\,bh, \text{ and } h = 2W/b.$$

But the height here is the desired F_s maximum. Hence,

$$F_s\ (\max) = \frac{2W}{b} = \frac{2 \times 450}{90} = 10 \text{ nt}$$

[b] CORRECT ANSWER: D

D - The only force acting on the particle is that of gravity, directed along the negative y-axis. Since, however, the net *vertical displacement of the sphere, at the point it hits the ground, is zero,* the net work done by the force of gravity is zero and, therefore, $\Delta K = 0$. Thus,

$$K_f = K_i = (1/2)mv_o^2 = (1/2) \times 2 \times (10)^2 = 100 \text{ j}$$

[c] CORRECT ANSWER: D

D - Given that

$$F = m(ax + b)^2$$

the work done is expressed by

$$W = \int_{x_1}^{x_2} \vec{F}\cdot d\vec{s} = m\int_{x_1}^{x_2} (ax + b)^2\,dx$$

$$= m\int_{x_1}^{x_2} (a^2x^2 + 2abx + b^2)\,dx$$

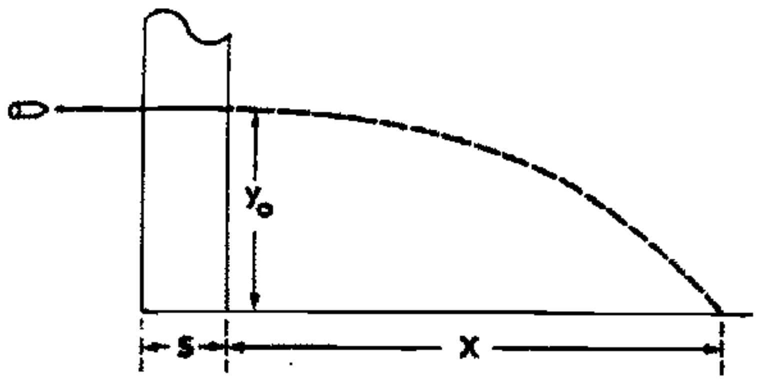
$$= m\left(a^2\,\frac{x^3}{3} + 2ab\,\frac{x^2}{2} + b^2x\right)\Bigg|_{x_1}^{x_2}$$

Set $x_1 = 0$ and $x_2 = 3$, to obtain

$$W = 78 \text{ j}$$

TRUE OR FALSE? The solution of this problem assumes the applied force to be constant over the entire interval from $x_1 = 0$ to $x_2 = 3$.

[a] CORRECT ANSWER: 128 m



We use the work-energy theorem to find the speed of the bullet when it comes out of the pole. (Note that the force is opposite to the direction of motion.)

$$-Fs = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2$$

or

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_o^2 - Fs = \frac{1}{2}(mv_o^2 - 2Fs)$$

and

$$v = \sqrt{\frac{(mv_o^2 - 2Fs)}{m}} \qquad (1)$$

The bullet comes out of the pole horizontally, so $v_{oy} = 0$. Also the final $y = 0$, so from

$$y = y_o + v_{oy}t - \frac{1}{2}gt^2$$

we find the time it takes the bullet to hit the ground

$$t = \sqrt{\frac{2y_o}{g}} \qquad (2)$$

Finally, since there is no acceleration in the x-direction,

$$x = v_{ox}t \qquad (3)$$

next page

continued

In equation (3), v_{ox} is the speed found in (1) and t is the time found in (2); thus,

$$x = \sqrt{\frac{(mv_o^2 - 2Fs)}{m}\frac{2y_o}{g}} \qquad (4)$$

Substituting the given data in (4) we find

$$x = \sqrt{\frac{[30 \times 10^{-3} \times (300)^2 - 2 \times 2500 \times 0.30]\ (2 \times 2)}{(30 \times 10^{-3})\ 9.8}}$$

$$= \sqrt{\frac{(2700 - 1500)\ 4}{0.294}} = 128 \text{ m}$$

TRUE OR FALSE? After passing through the pole, the trajectory of the bullet is parabolic.

[a] CORRECT ANSWER: D

D - The work done by a force $\vec{F}$ in moving a body through a displacement $d\vec{s}$ has been defined as

$$dW = \vec{F}\cdot d\vec{s} \qquad (1)$$

Dividing both sides of (1) by the time differential dt we find

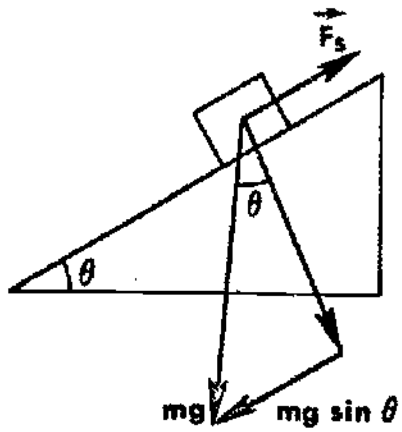
$$\frac{dW}{dt} = \vec{F}\cdot\frac{d\vec{s}}{dt} = \vec{F}\cdot\vec{v} \qquad (2)$$

But dW/dt is the rate at which work is done by the force; i.e., the power delivered by the force. Hence,

$$P = \vec{F}\cdot\vec{v} \qquad (3)$$

In the present problem we want to find the power when the mass is at the equilibrium point. From the relation $F = -k(x - x_o)$ we see that at $x = x_o$, $F(x_o) = 0$. Therefore, the spring delivers no power at the instant the mass m passes through the mid-point x_o.

[a] CORRECT ANSWER: 30°



The component of the applied force along the inclined plane, F_s, must at least equal the component of the weight along the incline. We have

$$F_s = mg\ \sin\theta$$

and

$$W = \vec{F}\cdot\vec{s} = F_s s$$

$$= (mg\ \sin\theta)\ s$$

so

$$\sin\theta = W/mgs$$

$$\theta = \sin^{-1}(W/mgs)$$

$$= \sin^{-1}[720/(3 \times 32 \times 15)]$$

$$= 30°$$

TRUE OR FALSE? The work required to move the block up the plane varies directly with the mass of the block.

[a] CORRECT ANSWER: B

B - A constant force produces a constant acceleration. Using the equation involving a, v, and s, with $v_o = 0$, we get

$$v^2 = 2as$$

or

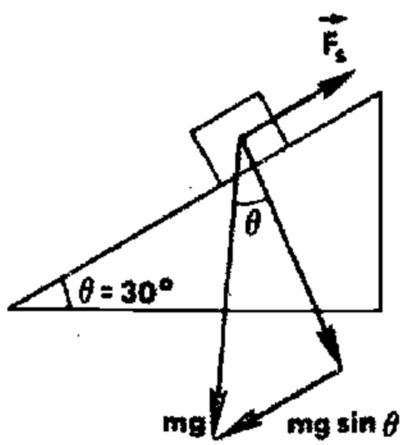
$$a = \frac{v^2}{2s}$$

The force is given by $F = ma$, thus $F = mv^2/2s$. The work done by this force is therefore

$$W = \vec{F}\cdot\vec{s} = Fs\cos 0° = \frac{mv^2}{2s}s = \frac{mv^2}{2}$$

[b] CORRECT ANSWER: 480 ft-lb

The component of the applied force along the inclined plane, F_s, must be equal and opposite to the component of the weight along the incline. Thus,



$$F_s = mg\sin\theta$$

and

$$W = \vec{F}\cdot\vec{s} = F_s s = (mg\sin\theta)\, s$$

$$= 2\text{ slugs} \times 32\frac{\text{ft}}{\text{sec}^2} \times \frac{1}{2} \times 15\text{ ft}$$

$$= 480\text{ ft-lb}$$

(We have used the fact that $1\text{ slug} = 1\,\frac{\text{lb-sec}^2}{\text{ft}}$)

TRUE OR FALSE? The work done in moving the safe a distance of 15 ft along the plane is independent of the angle of the plane.

[a] CORRECT ANSWER: A

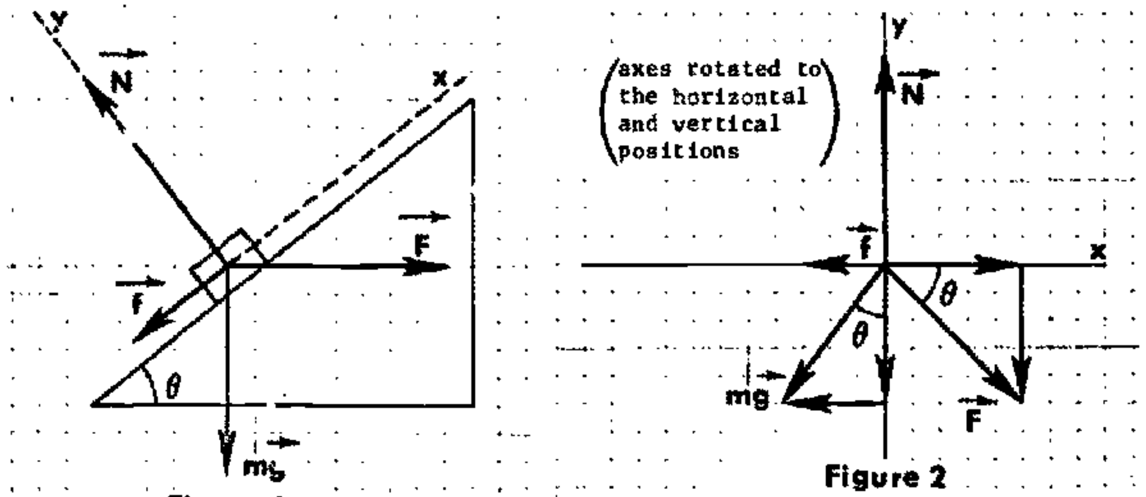


Figure 1 Figure 2

In Figure 1 we show all the forces acting on the block, along with the coordinate system chose for this problem. The forces involved are:

i) the applied horizontal force $\vec{F}$,

ii) the block's weight $m\vec{g}$,

iii) the plane's reaction force $\vec{N}$, and

iv) the force of friction $\vec{f}$ of magnitude $f = \mu N$.

Since the displacement is along the positive x-axis, only the x-components of these forces will contribute to the work done on the block. Therefore, the normal reaction force $\vec{N}$ will not contribute directly to the work done, being perpendicular to $\vec{s}$. It does contribute indirectly, however, since the magnitude of the frictional force $\vec{f}$ is proportional to $\vec{N}$. The direction of $\vec{f}$ is, of course, always opposite to the motion.

The force $\vec{N}$ is

$$\vec{N} = -\Sigma \vec{F}_y$$

or written in scalar form.

$$N = -(-F \sin\theta - mg \cos\theta) = F \sin\theta + mg \cos\theta$$

next page

continued

So the magnitude of the force of friction becomes

$$f = \mu N = \mu(F \sin\theta + mg \cos\theta) \qquad (1)$$

The total work done on the block is (see Figure 2)

$$W = \vec{F}\cdot\vec{s} = s\Sigma F_x = s[F \cos\theta - mg \sin\theta - \mu(F \sin\theta + mg \cos\theta)] \qquad (2)$$

Using the work-energy theorem $\Delta K = W$, we find

$$\frac{1}{2} mv^2 = W$$

or

$$s = \frac{1}{2} mv^2/[F \cos\theta - mg \sin\theta - \mu(F \sin\theta + mg \cos\theta)] \qquad (3)$$

$$s = \frac{\frac{1}{2}(10)(2.1)^2}{(120)(0.8) - (10)(9.8)(0.6) - (0.2)[(120)(0.6) + (10)(9.8)(0.8)]}$$

$$= 3.1 \text{ m}$$

TRUE OR FALSE? In this problem, the block comes to rest after traveling 3.0 m up the plane.

[a] CORRECT ANSWER: 3.6×10^5 ft-lb

In using the expression for the kinetic energy

$$K = \frac{1}{2} mv^2$$

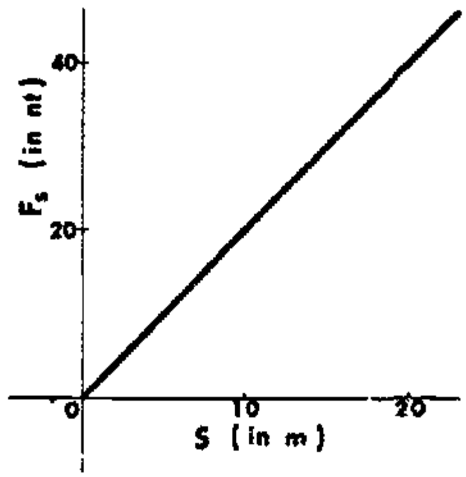
we must make sure that consistent units are used for all the quantities involved. Thus, $m = 3000/32$ slugs and $v = 60$ mi/hr $= 88$ ft/sec, so

$$K = \frac{1}{2}\left(\frac{3000}{32}\right)(88)^2 = 3.63 \times 10^5 \text{ ft-lb}$$

[a] CORRECT ANSWER: 300 j

The work done by a variable force is given by

$$W = \int_{s_1}^{s_2} \vec{F}\cdot d\vec{s} = \int_{s_1}^{s_2} F_s ds$$



From the graph we see that F_s depends linearly on s ($F_s = ks$), with the slope k equal to (40 nt)/(20 m) = 2 nt/m. Hence,

$$W = k\int_{s_1}^{s_2} s ds = (1/2)ks^2 \Big|_{s_1}^{s_2} = (1/2)k\ (s_2{}^2 - s_1{}^2)$$

$$= (1/2)(2)(400 - 100) = 300\ j$$

The work can also be computed "geometrically." The integral $\int F_s ds$ is equal to the area under the F_s versus s curve, between the specified s-limits. A look at the graph will convince you that the area is indeed 300 nt-m = 300 j.

[a] CORRECT ANSWER: 8 m/sec

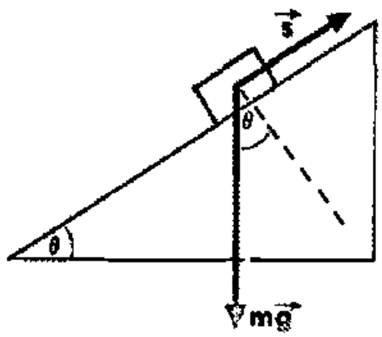
The work done by the block is equal to the change in kinetic energy K of the block.

$$W = \Delta K$$

or

$$W = K_f - K_i$$

All the work on the block is done by the component of weight projected along the incline. From the diagram, we have



$$W = m\vec{g}\cdot\vec{s}$$

$$= mgs\cos(\theta + 90°)$$

$$= -mgs\sin\theta$$

The work-energy theorem becomes

$$-mgs\sin\theta = \frac{1}{2}m(v_f^2 - v_i^2)$$

or

$$v_f^2 = v_i^2 - 2gs\sin\theta$$

Using the given data, $v_i = 10$ m/sec, $s = 2.6$ m, and $\theta = 45°$, we find $v_f = 8$ m/sec.

TRUE OR FALSE? This block will come to rest at the end of its upward journey after traveling a distance of 2.5 m up the plane.

[b] CORRECT ANSWER: C

C - The work-energy theorem states that the work done is equal to the change of the kinetic energy of the particle,

$$W = \Delta K = K_f - K_i \qquad (1)$$

From the given data,

$$K_i = (1/2)mv^2 \qquad (2)$$

and

$$K_f = (1/2)m(2v)^2 = 2mv^2 \qquad (3)$$

Substituting (2) and (3) into (1), we find

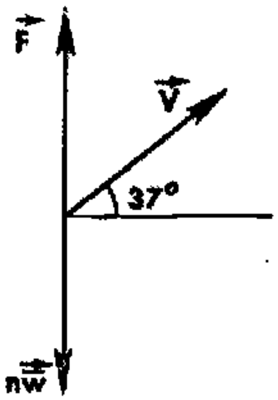
$$W = 2mv^2 - (1/2)mv^2 = (3/2)mv^2$$

[a] CORRECT ANSWER: A

A - The escalator must just overcome the gravitational force on the passengers (weight). If n is the number of passengers of average weight $\bar{W}$, the total force that must be provided by the escalator is $\vec{F} = -n\vec{\bar{W}}$. The power delivered by the escalator in moving the passengers with a velocity $\vec{v}$ is

$$P = \vec{F}\cdot\vec{v} = -n\vec{\bar{W}}\cdot\vec{v} = -n\bar{W}v\cos(90 + 37°)$$

$$= n\bar{W}v\sin(37°)$$



This power cannot be larger than

$$10\text{ hp} \times 550\,\frac{\text{ft-lb/sec}}{\text{hp}} = 5500\text{ ft-lb/sec}$$

Therefore, for maximum n,

$$n\bar{W}v\sin(37°) = 5500$$

or

$$n = \frac{5500}{\bar{W}v\sin(37°)}$$

$$= 30.6$$

Hence, 30 people can ride the escalator. Notice that 31 people are too many.

TRUE OR FALSE? In the solution above, power consumed is independent of the angle of the escalator.

[b] CORRECT ANSWER: D

Circular motion provides an example of the dependence of the work done on the angle between the applied force and direction of motion. The centripetal force is mv^2/r, and the total distance traveled during one revolution is $s = 2\pi r = \pi d$. At any moment, however, the force is directed along the radius toward the center, while the direction of motion is along the tangent to the circle at the point at which the particle is. Thus, the angle between $\vec{F}$ and $d\vec{s}$ is 90°; and $\vec{F}\cdot d\vec{s} = Fds\cos(90°) = 0$. So the work done by the centripetal force is zero.

TRUE OR FALSE? In uniform circular motion, the work done by the centripetal force is an unpredictable function of displacement.

[a] CORRECT ANSWER: 10 m/sec

The kinetic energy K of the projectile is given by

$$K = \frac{1}{2} m(v_x{}^2 + v_y{}^2)$$

At the highest point in a trajectory, the vertical component of velocity vanishes,

$$K = \frac{1}{2} mv_x{}^2$$

the horizontal component of velocity remains unchanged throughout the projectile's flight because there is no horizontal force. The initial speed, v, and v_x are related by

$$v_x = v \cos\theta$$

so that
$$K = \frac{1}{2} m(v \cos\theta)^2$$

or
$$v = \frac{1}{\cos\theta}\sqrt{\frac{2K}{m}}$$

$$= 10 \text{ m/sec}$$

TRUE OR FALSE? Just before the projectile leaves the muzzle of the gun it has definite vertical and horizontal components of velocity.

[b] CORRECT ANSWER: 100 ft-lb

Since the displacement and time are given, and $v_o = 0$, the constant acceleration can be computed from

$$s = \frac{1}{2} at^2 \qquad \text{so} \qquad a = \frac{2s}{t^2}$$

The force causing this acceleration is

$$F = ma = \frac{2ms}{t^2}$$

Finally the work done by this force is given by

$$W = Fs = 2m \frac{s^2}{t^2} = 2 \times \frac{16}{32} \times \left(\frac{100}{10}\right)^2 = 100 \text{ ft-lb}$$

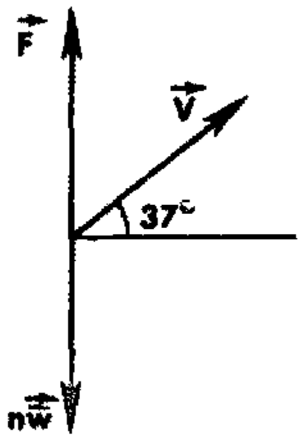
[a] This solution is identical to that of question 10, except for some numerical changes. The solution is reproduced here for your convenience.

CORRECT ANSWER: B

B - The escalator must just overcome the gravitational force on the passengers (weight). If n is the number of passengers of average weight $\vec{W}$, the total force that must be provided by the escalator is $\vec{F} = -n\vec{W}$. The power delivered by the escalator in moving the passengers with a velocity $\vec{v}$ is

$$P = \vec{F}\cdot\vec{v} = -n\vec{W}\cdot\vec{v} = -nWv\cos(90 + 37°)$$

$$= nWv\sin(37°)$$



This power cannot be larger than

$$5\text{ hp} \times 550\ \frac{\text{ft-lb/sec}}{\text{hp}} = 2250\text{ ft-lb/sec}$$

Therefore, for maximum n,

$$nWv\sin(37°) = 2250$$

or

$$n = \frac{2250}{Wv\sin(37°)}$$

$$= 30.6$$

Hence, 30 people can ride the escalator. Notice that 31 people are too many.

TRUE OR FALSE? More people could have ridden the escalator if W had been smaller than 150 lb.

[b] CORRECT ANSWER: D

D - The work done on the book is given by $W = \vec{F}\cdot\vec{s} = F_s s$ where $\vec{F}$ is the applied force and $\vec{s}$ is the displacement. In this question $\vec{s} = \vec{y}$. Furthermore, in order to move the book vertically upward (without acceleration), a vertical force $\vec{F} = m\vec{g}$ is required to overcome the force of "gravity" (weight). Hence, $F_y = mg$ and

$$W = \vec{F}\cdot\vec{s} = F_y y = mgy$$

[a] CORRECT ANSWER: D

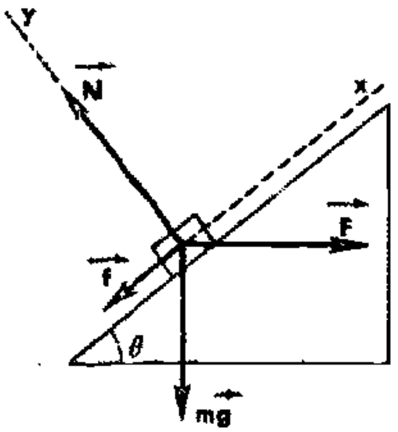


Figure 1

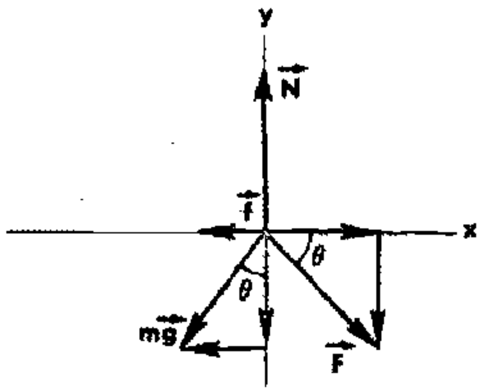


Figure 2

D - In Figure 1 we show all the forces acting on the block, along with the coordinate system chosen for this problem: The forces involved are:

i) the applied horizontal force $\vec{F}$,

ii) the block's weight $m\vec{g}$,

iii) the plane's reaction force $\vec{N}$, and

iv) the force of friction $\vec{f}$ of magnitude $f = \mu N$.

Since the displacement is along the positive x-axis, only the x-components of these forces will contribute to the work done on the block. Therefore, the normal reaction force $\vec{N}$ will not contribute directly to the work done, being perpendicular to $\vec{s}$. It does contribute indirectly, however, since the magnitude of the frictional force $\vec{f}$ is proportional to $\vec{N}$. The direction of $\vec{f}$ is, of course, always opposite to the motion.

The force $\vec{N}$ is

$$\vec{N} = -\Sigma \vec{F}_y$$

or written in scalar form,

$$N = -(-F \sin\theta - mg \cos\theta) = F \sin\theta + mg \cos\theta$$

next page

continued

So the magnitude of the force of friction becomes

$$f = \mu N = \mu(F \sin\theta + mg \cos\theta) \tag{1}$$

The total work done on the block is (see Figure 2)

$$W = \vec{F}\cdot\vec{s} = s(\Sigma F_x) = s[F \cos\theta - mg \sin\theta - \mu(F \sin\theta + mg \cos\theta)] \tag{2}$$

Using the work-energy theorem $\Delta K = W$, we find

$$\frac{1}{2} mv^2 = W$$

or

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2s}{m}[F \cos\theta - mg \sin\theta - \mu(F \sin\theta + mg \cos\theta)]} \tag{3}$$

Substituting the given data in (3) we obtain

$$v = \sqrt{\frac{2 \times 10}{10}\{(120)(0.8) - (10)(9.8)(0.6) - (0.2)[(120)(0.6) + (10)(9.8)(0.8)]\}}$$

$$= \sqrt{2(96 - 58.8 - 30.08)} = \sqrt{14.24} = 3.76 \text{ m/sec}$$

TRUE OR FALSE? With the axes chosen as illustrated, the y-component of F does not directly contribute to the work done on the block.

[a] CORRECT ANSWER: 100 J

At its highest altitude, the vertical component of the projectile's velocity is momentarily zero. The horizontal component of the velocity is constant, since there is no horizontal force acting on the particle. Hence, the kinetic energy at the highest altitude is given by

$$K = \frac{1}{2} mv_x{}^2 = \frac{1}{2} m(v_0 \cos\theta)^2 = \frac{1}{2} \times 2 \times (20 \times 0.5)^2 = 100 \text{ J}$$

TRUE OR FALSE? At the highest altitude, the projectile is accelerating uniformly in the horizontal direction.

[a] CORRECT ANSWER: C

C - The work done is found from

$$\int_{s_1}^{s_2} F_s ds \equiv \int_{x_1}^{x_2} F\, dx$$

with $F = -k(x - x_0)$. We have

$$W = -k\int_{x_1}^{x_2} (x - x_0)\, dx = -k\left(\frac{x^2}{2} - x_0 x\right)\Bigg|_{x_1}^{x_2}$$

$$= -\frac{k}{2}\,(x_2^{\,2} - 2x_0x_2 - x_1^{\,2} + 2x_0x_1)$$

Substituting the values of $x_0 = 0.15$ m, $x_1 = 0.10$ m, $x_2 = 0.25$ m, and $k = 2000$ nt/m above, we get

$$W = -1000(625 - 750 - 100 + 300) \times 10^{-4} = -7.5 \text{ j}$$

The negative sign means that work is done *on* the spring, not *by* the spring. We can see this qualitatively. The spring is originally compressed 5 cm from equilibrium. It does work in moving the mass from $x = 10$ cm to $x = x_0 = 15$ cm. From $x = 15$ cm to $x = x_2 = 25$ cm, however, the spring must be stretched, so work must be done on it.

TRUE OR FALSE? Throughout the displacement from $x = 18$ cm to $x = 23$ cm, the nature of the work done is identified by a negative (-) sign.

[b] CORRECT ANSWER: 980 watts

The power delivered by a force can be written as

$$P = \vec{F}\cdot\vec{v}$$

In the present problem, the magnitudes of the applied force and the force of friction μN are equal, so $F = \mu N = \mu mg$. Furthermore, $\vec{F}$ and $\vec{v}$ have the same direction. Hence,

$$P = Fv = \mu mgv = (0.2)(100)(9.8)(5) = 980 \text{ watts}$$

[a] CORRECT ANSWER: 5 cm

We use the work-energy theorem to find the speed of the ball when it comes out of the water. Note that the force is in the direction of motion.

$$W = \Delta K$$

$$\bar{F}s = \frac{1}{2} mv^2 - \frac{1}{2} mv_o^2$$

where $\bar{F}$ is the magnitude of the average force and s is the distance traveled by the ball in water. Solving for the square of the final speed, we have

$$v^2 = \frac{mv_o^2 + 2\bar{F}s}{m}$$

The speed v is now the initial speed with which the ball leaves the surface of the water. The flight of the ball through the air is described by

$$v_f^2 = v^2 - 2gy$$

At the highest point, $v_f = 0$, so

$$y = \frac{v^2}{2g} = \frac{mv_o^2 + 2\bar{F}s}{2mg} = \frac{\bar{F}s}{mg} \quad \text{where } v_o \text{ has been set equal to zero}$$

$$= \frac{(490 \times 10^{-5})\ (10)}{(0.1)\ (9.8)} = 0.05\ m = 5\ cm$$

TRUE OR FALSE? In this solution, $v_o = 0$ because the initial velocity of the ball is zero when it is released from the bottom of the barrel.

[b] CORRECT ANSWER: 10 m/sec

The work W done by the block is equal to the ch::ge in the kinetic energy K of the block,

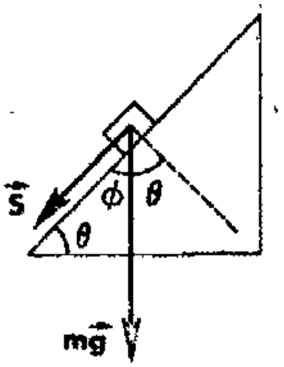
$$W = \Delta K$$

or

$$W = K_f - K_i$$

ne:: page

continued



In this case, we have work done by the component of weight projected along the incline, mg sinθ

$$W = m\vec{g}\cdot\vec{s}$$

$$= mgs\ \cos\phi$$

$$= mgs\ \sin\theta$$

The work-energy theorem becomes

$$mgs\ \sin\theta = \frac{1}{2}m(v_f^2 - v_i^2)$$

or

$$v_f^2 = v_i^2 + 2gs\ \sin\theta$$

Using the given data $v_i = 8$ m/sec, $s = 2.6$ m and $\theta = 45°$, we find $v_f = 10$ m/sec.

TRUE OR FALSE? All other things equal, at increasingly larger values of $\vec{s}$, the greater will be v_f.

[a] CORRECT ANSWER: C

Given that

$$F = ma = m(\alpha + \beta x^2)$$

the work done is

$$W = \int_{x_1}^{x_2} \vec{F}\cdot d\vec{s} = m\int_{x_1}^{x_2} (\alpha + \beta x^2)\ dx = m[\alpha x + \left(\frac{\beta}{3}\right) x^3] \Big|_{x_1}^{x_2}$$

$$= 2[5x + (15/3)\ x^3] \Big|_0^2 = 100\ J$$

TRUE OR FALSE? For th situation described, α and β are both constants.

[a] CORRECT ANSWER: B

B - The change in the block's kinetic energy is equal to the net work done on the block. The spring applies a force on the block up to the point the spring reaches its normal (equilibrium) length. This variable force is equal to

$$F = -k(x - x_0) \tag{1}$$

which is in the direction of the block's motion.

The work done by this force is

$$W_1 = \frac{1}{2} k(x - x_0)^2 \equiv \frac{1}{2} ks_1{}^2 \tag{2}$$

where s_1 is the change in the spring's length; namely,

$$x - x_0 = s_1 = -20 \text{ cm} = -0.2 \text{ m}$$

the minus sign indicating a negative change of length, that is, compression. The work done by the force of friction is negative (work done against friction) and is equal to

$$W_2 = -\mu mgs_2, \text{ with } s_2 = \overline{AB} = 50 \text{ cm} = 0.5 \text{ m}. \tag{3}$$

Thus, at point B the block's kinetic energy is given by

$$\frac{1}{2} mv^2 = W_1 + W_2 = \frac{1}{2} ks_1{}^2 - \mu mgs_2 \tag{4}$$

Now, since to the right of B the surface is frictionless, the net force on the block there is zero, and the block will move with constant velocity. Therefore, the block's speed at point C is found from equation (4) and is

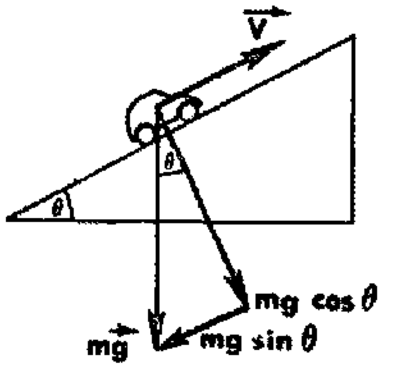
$$v = \sqrt{\frac{k}{m} s_1{}^2 - 2\mu gs_2} \tag{5}$$

Substituting the given data in (5) we find

$$v = \sqrt{\frac{573}{2} (0.2)^2 - 2(0.25)(9.8)(0.5)}$$

$$= \sqrt{11.45 - 2.45} = 3.00 \text{ m/sec}$$

(a) CORRECT ANSWER: 75,350. ft-lb/sec or 137 hp



The power can be computed from

$$P = \vec{F}\cdot\vec{v} \qquad (1)$$

The force $\vec{F}$ must be such as to overcome the weight of the car; i.e.,

$$\vec{F} = -m\vec{g} \qquad (2)$$

So,

$$\begin{aligned} P &= -m\vec{g}\cdot\vec{v} = mgv\cos(90+\theta) \\ &= -mgv(-\sin\theta) \\ &= mgv\sin\theta \qquad (3) \end{aligned}$$

Equation (3) can also be derived by using the fact that mg sinθ is the component of the weight along the incline. The force provided by the car's engine must overcome this component. The component of the weight normal to the incline (mg cosθ) is counteracted by the road's "reaction" on the car.

The given speed is 60 mi/hr = 88 ft/sec. Also, in order to convert from ft-lb/sec to hp, we must divide (3) by 550 (1 hp = 550 ft-lb/sec). Thus,

$$P = \frac{(2500)(88)(0.342)}{550} = 137 \text{ hp}$$

[a] CORRECT ANSWER: 825 lb

The work-energy theorem can be used for solving this problem. From $W = \Delta K$ or $\bar{F}s = (1/2)mv^2$, we find

$$\bar{F} = \frac{mv^2}{2s} \qquad (1)$$

where $\bar{F}$ is the average force, s the distance over which $\bar{F}$ acts, and v is the initial velocity of the bullet. The quantities involved in equation (1) must be expressed in the appropriate units. We convert the bullet's weight into its mass

$$m = 1 \text{ oz} \times \frac{1}{16}\,\frac{\text{lb}}{\text{oz}} \times \frac{1}{32}\,\frac{\text{slug}}{\text{lb}} = \frac{1}{16 \times 32}\text{ slug}$$

The velocity v = 600 mi/hr = 880 ft/sec, and the penetration depth s = 11 inches = 11/12 ft. Thus,

$$\bar{F} = \frac{\frac{1}{16 \times 32} \times (880)^2}{2 \times 11/12} = \frac{(88)^2 \times 12 \times 10^2}{16 \times 32 \times 2 \times 11} = 825 \text{ lb}$$

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL Potential Energy and Conservative Forces

OBJECTIVE

To introduce the concept of potential energy and its place in conservative systems; to use these concepts in the solution of relevant problems.

Potential energy is the kind of energy that a body has by virtue of its position or configuration. When a body is raised to a higher level, it is able to do a certain amount of work when it falls back to its original height; this is potential energy of "position." Potential energy of configuration is illustrated by a stretched or compressed spring. Work must be done to change the configuration of the spring so that an equal amount of potential energy is stored in the spring under ideal conditions because it can do that amount of work in returning to its original length.

In either case, potential energy can be uniquely determined only by referring the change of position or the change of configuration to some particular reference which is arbitrarily assigned a value of zero. *Only a change of potential energy is significant.* Once, however, a position is determined to which a zero value of potential energy can be assigned, we may speak of the potential energy of a body. When a mass is lifted to a table top, the energy may be measured relative to the floor. If we had referred the energy to the cellar or attic, the potential energy at floor level, or at table level, would then have been different.

In its broadest sense, a force is conservative if the work it does in moving an object over any path back to its starting point is zero. A second important characteristic of the action of a conservative force is that the work it does in moving the object between two given points is independent of the path over which the object has been moved. Conservative forces are intimately related to potential and kinetic energy. When a conservative force does work on a body, this work is completely recoverable; indeed, this is the fundamental aspect of conservative systems.

In this section, you will be asked to

(a) recognize the differences between conservative and non-conservative forces;

(b) extend the work-energy theorem to situations involving conservative forces.

PROBLEMS

1. The work-energy theorem states that the work done by the resultant force on a particle is equal to the change in kinetic energy of the particle, $W = \Delta K$. *If* the resultant force is conservative, we also know that the total energy of the particle does not change, $\Delta K + \Delta U = 0$. In this case, which of the following statements is correct?

The work done by the resultant conservative force is equal to

A. the change in the potential energy of the particle, $W = \Delta U$

B. the change in the total energy of the particle, $W = \Delta E$

C. the negative of the change in the total energy of the particle, $W = -\Delta E$

D. the negative of the change in the potential energy of the particle, $W = -\Delta U$

2. Which of the following describes the action of a conservative force?

A. A block of wood slides down an inclined plane with uniform speed.

B. A cork is pushed under water and then released to bob up to the surface.

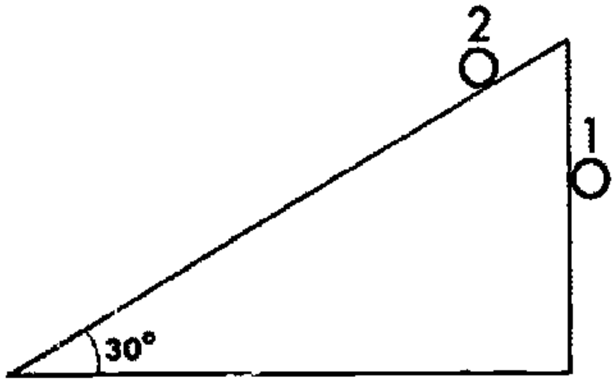
C. A meteor enters the atmosphere at high speed and succeeds in reaching the ground without burning up.

D. A rock is thrown vertically upward from the surface of the moon and allowed to fall back down.

3. Let the velocities of a particle at positions x_0 and x be v_0 and v, respectively. If the total energy E at x_0 is $(1/2)mv_0^2 + U(x_0)$ and the particle is subjected to a conservative force, then for the position x, the expression $(1/2)mv^2 + U(x)$ is the equivalent of

A. $E + W$
B. $E + \Delta U$
C. $E + \Delta K$
D. E

4.



Two particles of equal mass are rel[illegible]sed from top of an incline making an angle of 30° with the horizontal. Particle one falls straight down and particle two slides down the incline, both reaching the same zero level. Neglecting friction, find the ratio of the work done by the gravitational force on particle one to the work done by the gravitational force on particle two.

A. 0.5
B. 2
C. 1
D. 0.866

INFORMATION PANEL

Conservation of Energy

OBJECT[illegible]E

To apply the principle of the conservation of energy to the solution of numerical problems in which it is directly involved.

The pri[illegible]iple (or law) of conservation of energy states that in any isolated system, regardless of the changes that may occur within the system, the total energy of the system remains constant. Constancy implies that energy may be transferred from one part of the system to another, but that it cannot be created or destroyed. We know of no energy generators, only energy converters. The destruction of en[illegible]gy appears to be impossible. When energy seems to disappear, we always find that it has been merely transferred elsewhere and can always be accounted for.

next page

continued

In systems where *only conservative forces act*, the conservation principle may be written as:

$$K + U = \text{constant}$$

in which K = kinetic energy, U = potential energy, and the constant is called the total mechanical energy of the system. Any loss of kinetic energy that occurs results in a gain of an equal amount of potential energy, and vice versa so that

$$-\Delta U = \Delta K$$

$$\text{thus} \quad \Delta K + \Delta U = 0$$

When one or more nonconservative forces are present, the total mechanical energy E of the system is *not constant*, but changes by an amount equal to the work of the nonconservative force on the system.

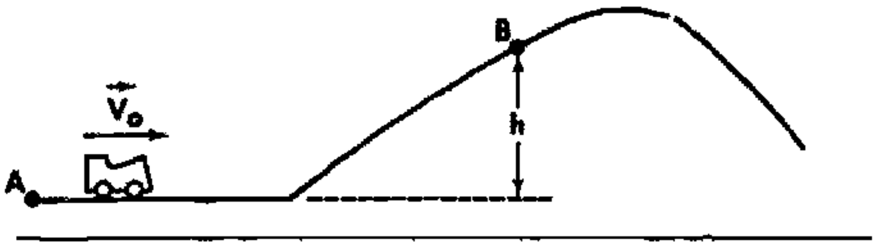
The problems associated with this section are all based on the conservation of mechanical energy, hence involve only conservative forces. You may assume for these that nonconservative forces are absent or that their effects may be ignored. You will be expected to be able to determine

(a) the maximum height of ascent of a vertically projected body with the help of the conservation principle;

(b) the height to which a roller coaster with a given initial speed will rise as it climbs an incline;

(c) determine the speed of a given pendulum bob as it passes through the lowest point of its swing.

5. A roller coaster moves at point A with speed v_o. At point B, the coaster moves with speed (1/2) v_o. Assuming no frictional losses, what is the height of point B above point A?



A. $3\ v_o^2/8g$

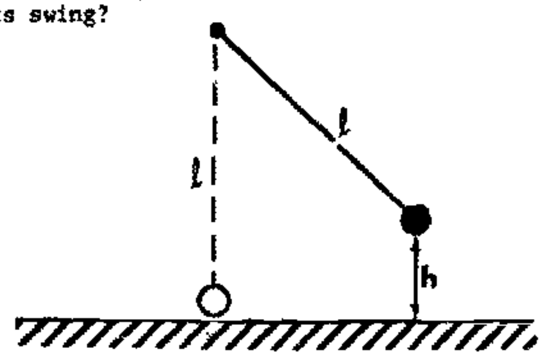
B. $7\ v_o^2/8g$

C. $v_o^2/4g$

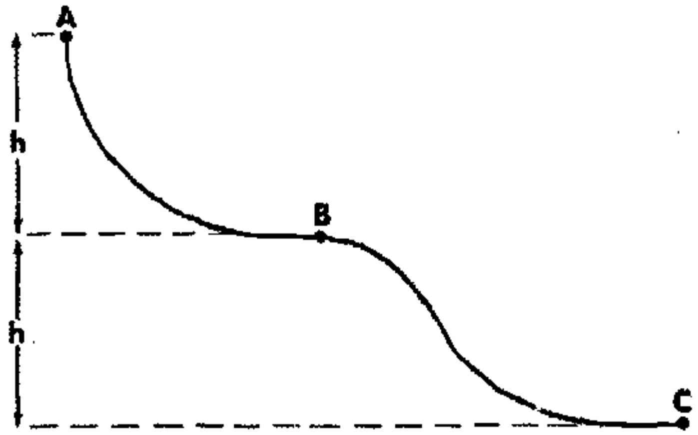
D. $5\ v_o^2/8g$

6. A ball of mass 0.5 kg is thrown from ground level vertically upward with a speed of 20 m/sec. Use conservation of energy to find the maximum height, h, attained by the ball.

7. A pendulum bob is released from a height h = 30 cm with a speed v_o = 2 m/sec. What is the speed of the bob when it passes through the lowest point of its swing?



8. A particle of mass m starts from rest at point A and slides down to point C without leaving the track. Neglecting friction, find the ratio of the speed of the particle at point B to its speed at point C, if the height of B above C is h and the height of A above C is 2h.



A. $\sqrt{2}$
B. 2
C. $\sqrt{1/2}$
D. 1/2

INFORMATION PANEL <u>Potential Energy When the Resultant Force Varies</u>

OBJECTIVE

To solve potential energy problems in which the force that produces the change in potential energy varies with the configuration of the system.

In one dimension, say, along the x-axis, the change in potential energy ΔU is related to the component of force along this axis $F(x)$ by the equation:

$$\Delta U = -\int_{x_0}^{x} F(x)\, dx$$

This relationship may also be written in the form:

$$F(x) = -\frac{d}{dx} U(x)$$

next page

continued

The second form is easily verified by substituting it in the first equation: an identity is obtained. Clearly, we can also write this last expression as:

$$dU(x) = -F(x)\ dx$$

In many situations, the force F varies with its position along the chosen axis, that is, F is a function of x. When you encounter a problem of this type, all that is required is that you set up the general equation you need and substitute the given identity for F(x). For example, suppose that you are told that the force is related to its position along the x-axis by:

$$F = kx^3$$

in which k is a constant. To find the change in the potential energy of a particle to which this force is applied, from some reference position x_0 to a new position x, you would write:

$$dU = -(kx^3)\ dx$$

and then integrate between the positions x and x_0.

Generalized expressions for forces that vary in two and three dimensions are also derived in your text. In this section of your work, however, you will be dealing with one-dimensional problems only. You are expected to be able to solve problems in which you are to find

(a) the potential energy of a particle located at some arbitrary point on the axis being considered, given the way in which the force varies with position on this axis;

(b) the x-component of a force that varies in two dimensions, given the relationship between this force and the consequent potential energy it produces.

9. For a force

$$F = -ky$$

where k is a constant, and for the choice U = 0 at $y = y_0$, what is the potential energy U(y) of a particle located at an arbitrary point y?

10. The potential energy corresponding to a certain two-dimensional force is

$$U(x,y) = \frac{1}{3} k(x^3 + y^3)$$

where k is a constant. The x-component of the corresponding force is

A. $-k(x^2 + y^2)$
B. kx^2
C. $-kx^2$
D. $k(x^2 + y^2)$

11. A particle is subject to a force $F(y) = -mg$. If the potential energy of the particle is zero at the origin, $U(0) = 0$, what is the potential energy $U(y)$ of the particle as a function of y?

12. For a force

$$F(x) = -\frac{k}{x^2}$$

where k is a constant and for the choice of $U = 0$ at $x = \infty$, what is the potential energy $U(x)$ of a particle located at an arbitrary point x?

A. $-\frac{k}{x}$ B. $\frac{k}{x^2}$ C. $\frac{k}{x}$ D. kx^2

INFORMATION PANEL — Energy in Springs

OBJECTIVE

To apply conservation principles to the solution of spring problems in which kinetic energy and potential energy are involved in ideal (frictionless) systems.

The problems found in this section require that you recall and apply the following fundamental relationships:

next page

continued

1. In an oscillating spring system isolated from external forces, the total energy remains constant and is always instantaneously equal to the sum of the kinetic energy and potential energy of the system at that instant, or:

$$E = K + U$$

2. The force required to compress or stretch a spring is proportional to the compression or extension, i.e.,

$$F_{app} = kx.$$

where k is defined as being the spring constant. The potential energy of the spring is therefore

$$U = \frac{1}{2} kx^2$$

in which k is the spring constant and x is the displacement of the end of the spring from its zero reference position.

3. When a spring is compressed or stretched as a result of the transfer of energy to it from a moving mass m having a velocity v, the conservation principle may be written:

$$E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2$$

where E is the constant, total energy of the mass-spring system.

In this section, you will be asked to solve problems in which you must determine:

(a) the height from which a mass must be dropped onto a spring in order to produce a given compression;

(b) the kinetic energy of a mass on a vibrating spring when the total energy and potential energy at a given instant are known;

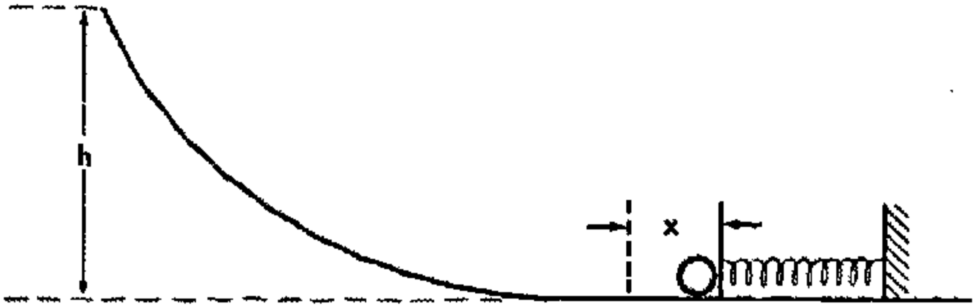
(c) the maximum displacement of a mass on a vibrating spring given the spring constant and the total energy of the system;

(d) the maximum compression of a given spring after being struck by a given mass moving at a given speed.

13. A ball of mass m is dropped from rest onto a spring with spring constant k. The maximum compression of the spring is x. Find the height above the (uncompressed) spring from which the ball was dropped, assuming no friction at the time of impact.

A. $(kx^2/2mg) - x$
B. $(kx^2/2mg) + x$
C. $(kx^2/mg) - x$
D. kx^2/mg

17. A particle of mass m is released from the top of an incline as shown in the figure below. At the bottom, it compresses a spring by an amount indicated as x. The spring constant is k. Find the height h from which the particle is released. Neglect function.



A. $\frac{2mg}{kx^2}$ B. $\frac{kx^2}{2mg}$ C. $\frac{kx^2}{mg}$ D. $\frac{mg}{kx^2}$

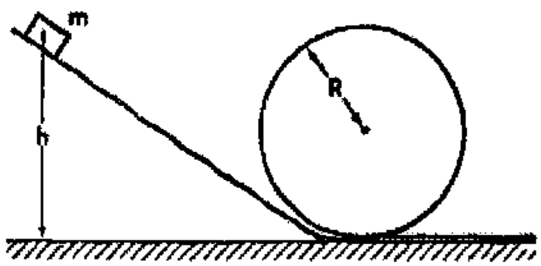
INFORMATION PANEL A Composite Problem Using Conservation of Energy

---

OBJECTIVE

To solve a problem in which the principle of conservation of energy is combined with centripetal force.

In the system illustrated in the accompanying diagram, a block is placed on a frictionless inclined track and released so that it slides down the track into an inside loop. If it is not placed high enough above the reference surface, it will start the loop but fall off before it reaches the top. If it is to successfully negotiate the loop and continue on its way, there is a definite minimum height at which it must be placed before it is released. Let's talk about this problem with a view to helping you get started on it.



You are given the mass or weight of the block and the radius of the loop. When the block passes the bottom of the loop just as it comes off the incline, it is moving at some specific speed, but the speed must decrease as it begins to climb the far end of the loop against gravity. The loop

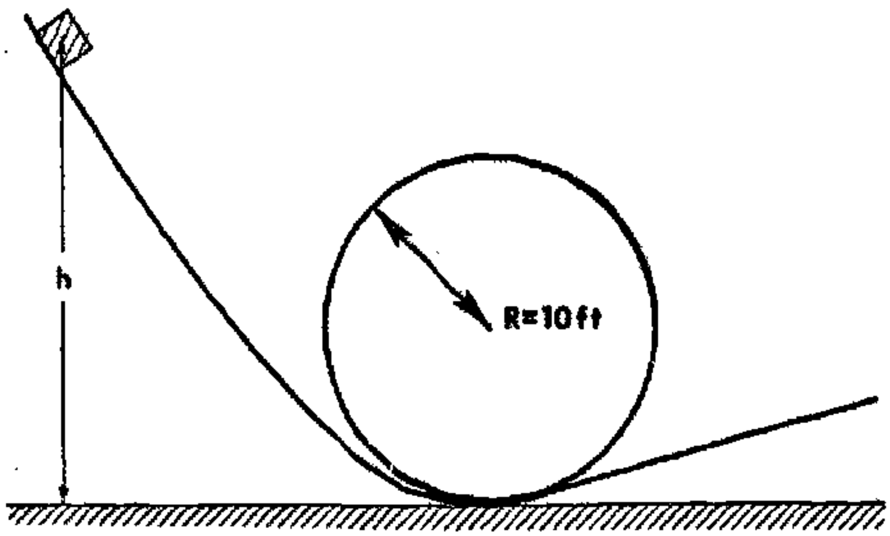
next page

continued

is rigid and presses inward on the block, providing the centripetal force needed to force it into circular motion. The lowest velocity to which the block will be reduced is the velocity at the very top of the loop. What physical situation must obtain at this point if the block is not to leave the track, falling downward? Think about this before continuing.

If the velocity of the block at the top of the loop is less than a certain critical value, its weight will be greater than the centripetal force required to keep it moving in a loop of the given radius. In this case, it will simply leave the track and fall back to the surface. Thus, if it is not to lose contact with the track, its velocity must be greater than this critical value. Stated otherwise, *its velocity must be such that the centripetal force needed to make it move in a circle of that particular radius is equal to or greater than the weight of the block.* This is all the clue you should need to get started. Set up the expression for centripetal force in terms of mass and velocity, then equate this with the weight expressed in terms of mass and gravitational acceleration. Look for a way to get the kinetic energy of the block into the picture after you have done this.

18.



Compute the minimum height h from which a 10-lb block can be released, in order that it will go around the loop without losing contact with the track. Assume a frictionless track.

[a] CORRECT ANSWER: 0.78 j

The system is conservative and the principle of conservation of energy holds:

$$E = K + U$$

where total energy E is a constant. Substituting the data 1.28 j = K + 0.50 j, we obtain K = 0.78 j.

[b] CORRECT ANSWER: 3.14 m/sec

If we take the lowest point as the zero-potential energy point, the bob's total energy at release is $E_i = (1/2)mv_o^2 + mgh$. This must be equal to its total energy when it passes through the lowest point of its swing, $E_f = (1/2)mv^2$. Equating E_i and E_f and solving for v, we obtain

$$\frac{1}{2} mv^2 = \frac{1}{2} mv_o^2 + mgh$$

or

$$v = \sqrt{v_o^2 + 2gh}$$

$$= \sqrt{(2)^2 + 2(9.8)(0.3)}$$

$$= 3.14 \text{ m/sec}$$

Notice that this is the result when the pendulum bob is initially moving either clockwise or counterclockwise.

[c] CORRECT ANSWER: C

The gravitational force is conservative, therefore, the work done is equal to negative of the change in potential energy. In other words,

$$\Delta W = -\Delta U$$

In this problem the ΔU is the same for both the particles, therefore, the ratio of the work done is *one*.

TRUE OR FALSE? To arrive at the statement W = -ΔU, one must apply *both* the principle of conservation of energy and the work-energy theorem.

[a] CORRECT ANSWER: A

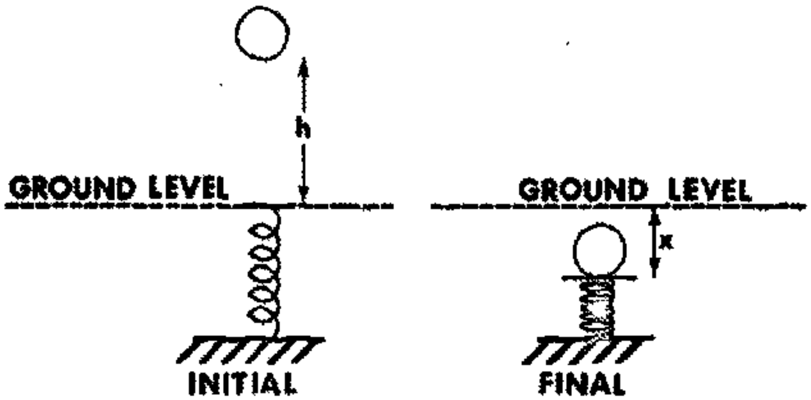
Initially, the kinetic energy of the ball is zero. If we define our zero of gravitational potential energy to be the "ground" level in the accompanying diagram, the potential energy of the ball is mgh, where h is the unknown height above the spring. The uncompressed spring has zero potential energy. Therefore, the total initial energy is

$$E_i = U_i + K_i = mgh + 0 = mgh \qquad (1)$$

In the final configuration, the ball is momentarily at rest (zero kinetic energy). The potential energy due to gravity is mg(-x) because the ball is below the "ground" level. The spring now contributes a potential energy of $(1/2)kx^2$ due to its compression. We have

$$E_f = U_f + K_f = [mg(-x) + (1/2)kx^2] + 0$$

$$= -mgx + (1/2)kx^2 \qquad (2)$$



Conservation of energy, $E_i = E_f$, gives

$$mgh = -mgx + (1/2)kx^2$$

or

$$h = \frac{kx^2}{2mg} - x$$

TRUE OR FALSE? As it turns out, the height from which the ball is dropped is directly proportional to the compression of the spring.

[a] CORRECT ANSWER: B

Applying the law of conservation of energy:

$$\frac{1}{2} mv^2 = mgh \qquad (1)$$

where v is the speed of the particle on the flat part of the track. Similarly, the energy relationship for the particle and spring system is:

$$\frac{1}{2} mv^2 = \frac{1}{2} kx^2 \qquad (2)$$

Combining (1) and (2) and solving for h yields

$$h = \frac{kx^2}{2mg}$$

TRUE OR FALSE? After the ball passes the bottom of the incline and while it moves along the flat part of the track, its acceleration is zero.

[b] CORRECT ANSWER: D

The force of gravity is a conservative force. Since the moon has no atmosphere, friction is not operative during the rock's flight. When the rock returns to the height from which it was thrown, it will have its initial speed and the kinetic energy will also have its initial value.

In general we look for dissipative forces (usually frictional forces), which will convert some, or all, of the mechanical energy into thermal energy (heat), light, or sound which, for practical purposes, is not recoverable. Such forces are non-conservative.

When a rock is thrown near the surface of the Earth, the process is not conservative unless air resistance is neglected. Although the gravitational force is conservative, the force due to friction is dissipative and the resultant force is therefore dissipative. Answers A, B, and C involve frictional forces and are therefore not descriptive of conservative forces.

[a] CORRECT ANSWER: C

Using the principle of conservation of mechanical energy, the speed at points B (v_B) and the speed at point C (v_C) may be computed from

$$\frac{1}{2} m v_B^2 = mgh \qquad (1)$$

and

$$\frac{1}{2} m v_C^2 = mg(2h) \qquad (2)$$

Therefore the ratio

$$\frac{v_B}{v_C} = \sqrt{\frac{1}{2}}$$

TRUE OR FALSE? At point B, the kinetic energy is smaller than the total energy of the particle at that instant.

[b] CORRECT ANSWER: D

The forces are conservative, so the total energy at x is equal to the total energy at x_0. Hence,

$$\text{Total energy at } x_0 = \frac{1}{2} m v_0^2 + U(x_0) = E$$

so that

$$\text{total energy at } x = \frac{1}{2} m v^2 + U(x) = E$$

Hence, the total energy at x = E. Clearly, answers A, B, C all have an incorrect additive on the right side of the equation.

[c] CORRECT ANSWER: D

When conservative forces are involved, the total energy is conserved. Thus, $\Delta E = \Delta K + \Delta U = 0$, and $\Delta K = -\Delta U$. Then, using the work-energy theorem, we get,

$$W = \Delta K = -\Delta U$$

TRUE OR FALSE? The statement $\Delta K + \Delta U = 0$ is the equivalent of the statement $K + U =$ constant for a conservative system.

[a] CORRECT ANSWER: 20.4 m

We choose our zero level of potential energy at the position from which the ball was thrown. Its total energy at the instant it leaves the ground consists entirely of its initial kinetic energy, $(1/2)mv_o^2$. Similarly, at the instant the ball attains its maximum height its total energy is potential, mgh. Using conservation of energy we obtain

$$mgh = \frac{1}{2} mv_o^2$$

which gives

$$h = \frac{v_o^2}{2g} = \frac{(20)^2}{2 \times 9.8} = \frac{400}{19.6} = 20.4 \text{ m}$$

In general, for a single particle in a gravitational field, equating initial energy to final energy yields

$$mgy_o + \frac{1}{2} mv_o^2 = mgy + \frac{1}{2} mv^2$$

from which

$$v^2 = v_o^2 - 2g(y - y_o)$$

This equation should be familiar to you from kinematics.

[b] CORRECT ANSWER: 0.8 m

At either end of the swing ($x = x_{max}$) the speed of the mass is momentarily zero, so the total energy of the mass-spring system (1.28 j) is potential. Thus,

$$U = \frac{1}{2} k(x_{max} - x_o)^2 = E = 1.28 \text{ j}$$

and the distance from the equilibrium position

$$s = x_{max} - x_o = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(1.28)}{4}} = \sqrt{0.64} = 0.8 \text{ m}$$

[a] CORRECT ANSWER: 25 ft

At the top of the loop the block must have a velocity such that the centripetal force required to make it move in a circle is at least as great as the weight of the block. It can, of course, be greater, since the track can provide all the additional force required. We must, then, have

$$\frac{mv^2}{R} \geq mg$$

or

$$\frac{mv^2}{2} \geq \frac{mgR}{2} \qquad (1)$$

But from conservation of energy (no friction)

$$\frac{mv^2}{2} = mg(h - 2R) \qquad (2)$$

Combining (1) and (2) we obtain

$$mg(h - 2R) \geq \frac{mgR}{2}$$

and

$$h \geq 2R + \frac{R}{2} = \frac{5R}{2}$$

or

$$h \geq 25 \text{ ft}$$

TRUE OR FALSE? When the block is at the highest point of the loop, the centripetal force must be greater or equal to the weight of the block.

[a] CORRECT ANSWER: B

The initial state consists of a moving block with kinetic energy $(1/2)mv^2$, and an uncompressed spring with zero potential energy.

The total energy initially is

$$E_i = \frac{1}{2} mv^2$$

The final configuration consists of the block momentarily at rest, and the spring compressed an unknown distance x. We have

$$E_f = \frac{1}{2} kx^2$$

Now since energy is conserved, $E_i = E_f$, which gives

$$x^2 = \frac{m}{k} v^2 \quad \text{or} \quad x = \sqrt{\frac{m}{k}}\, v$$

[b] CORRECT ANSWER: A

Mechanical energy is conserved. Taking the initial potential energy to be zero, we have $E = (1/2)mv_o^2$. Equating this to the total final energy, kinetic plus potential, we obtain

$$\frac{1}{2} mv_o^2 = \frac{1}{2} m\left(\frac{v_o}{2}\right)^2 + mgh$$

The resulting expression for h is

$$h = \frac{(1/2)v_o^2 - (1/8)v_o^2}{g} = \frac{3v_o^2}{8g}$$

TRUE OR FALSE? The initial total energy of the roller coaster is partly kinetic and partly potential.

[a] CORRECT ANSWER: $U(y) = mgy$

In one dimension we have

$$F(y) = -\frac{dU(y)}{dy}$$

or

$$dU = -F(y)\,dy \qquad (1)$$

When integrating equation (1) remember that

$$\int_{U(0)}^{U(y)} dU = -\int_0^y F(y)\,dy = \int_0^y mg\,dy$$

or

$$U(y) - U(0) = -mgy\Big|_0^y = mgy$$

Since $U(0) = 0$,

$$U(y) = mgy$$

[b] CORRECT ANSWER: C

The force in two dimensions is given by

$$\vec{F} = -\frac{\partial U}{\partial x}\vec{i} - \frac{\partial U}{\partial y}\vec{j} \qquad (1)$$

In the computation of $\partial U/\partial x$, y is treated as though it were a constant. We have

$$F_x = -\frac{\partial}{\partial x}\left[\frac{1}{3}k(x^3 + y^3)\right]$$

but since $\partial x^3/\partial x = d(x^3)/dx = 3x^2$

and $\partial(y^3)/\partial x = 0$

then $F_x = -\frac{1}{3}k(3x^2) = -kx^2$, the answer to the problem.

[a] CORRECT ANSWER: $\frac{1}{2}k(y^2 - y_o^2)$

From
$$F_y = -\frac{\partial U}{\partial y} = -\frac{dU(y)}{dy}$$

we find
$$dU = -F\,dy$$

Integrating between the points y_o and y,
$$\int_{U(y_o)}^{U(y)} dU = -\int_{y_o}^{y} F\,dy$$

Using the given data $F = -ky$ and $U(y_o) = 0$, we find
$$U(y) = \frac{ky^2}{2}\Bigg|_{y_o}^{y} = \frac{1}{2}k(y^2 - y_o^2)$$

TRUE OR FALSE? According to our established symbolism, the force F in this problem is confined to one dimension.

[b] CORRECT ANSWER: C

From
$$F(x) = -\frac{\partial U}{\partial x} = -\frac{dU}{dx}$$

we find
$$dU = -F(x)\,dx = \frac{k}{x^2}\,dx.$$

Integrating from the points $x = \infty$ to $x = x$
$$\int_{U(\infty)}^{U(x)} dU = \int_{\infty}^{x} \frac{k}{x^2}\,dx$$

$$U(x) = \frac{k}{x}$$

Note that $U(\infty) = 0$

TRUE OR FALSE? The potential energy of the particle as x approaches ∞ is less than k but greater than zero.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL

The Coordinates of the Center of Mass of a System of Particles

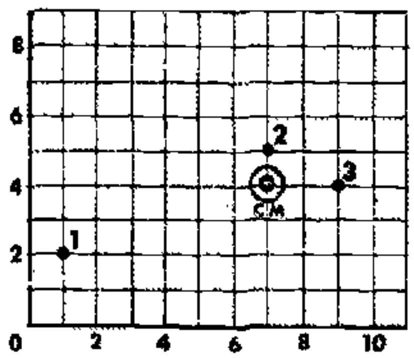
OBJECTIVE

To recognize and apply the fact that the position of the center of mass of a system of particles is independent of the coordinates used to describe its location.

Your assigned reading will ultimately carry you to the statement:

> *The center of mass of a system of particles depends only on the masses of the particles and the position of the particles relative to one another.*

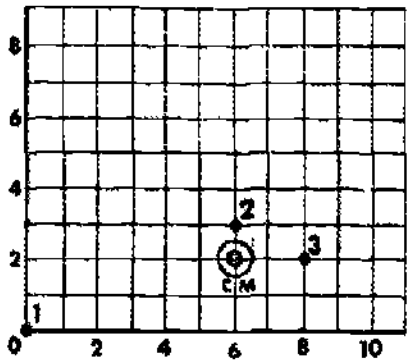
One important implication of this statement is that the orientation or position of the axes does not affect the position of the center of mass relative to the particles. Consider the three particles in the upper diagram at the left. These particles have relative masses of 1, 2, and 3 respectively as indicated. Using the accepted method for determining the coordinates of the center of mass, we can write:



$$\bar{x}_{cm} = \frac{(1)(1) + (2)(7) + (3)(9)}{1 + 2 + 3} = 7$$

$$\bar{y}_{cm} = \frac{(1)(2) + (2)(5) + (3)(4)}{1 + 2 + 3} = 4$$

In the lower diagram, we have redrawn the system with a new set of axes so that the particle of relative mass 1 is now at the origin. Once again writing the coordinates of the center of mass, we have



$$\bar{x}'_{cm} = \frac{(1)(0) + (2)(6) + (3)(8)}{1 + 2 + 3} = 6$$

$$\bar{y}'_{cm} = \frac{(1)(0) + (2)(3) + (3)(2)}{1 + 2 + 3} = 2$$

next page

continued

Compare the two diagrams and observe that the center of mass in each one is identically located *relative to the system of particles.* Despite the fact that the particle of mass 1 is squarely on the origin and so has coordinates of (0,0), it still enters into the calculation of the center of mass. Note that it *does* contribute to the denominator of the fraction although it drops out of the numerator in both the evaluations.

To handle the problems in this group, it will be necessary for you to

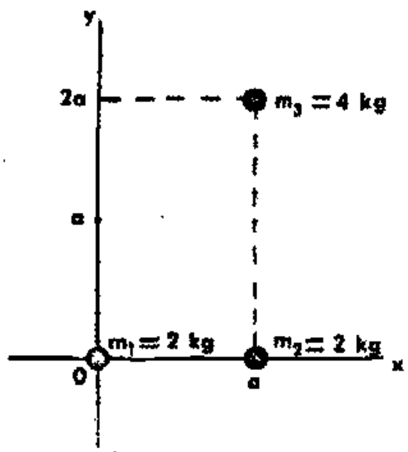
(a) locate the center of mass of three particles, two of which lie on the x-axis;

(b) write an expression for the x-coordinate of the center of mass of two particles in general form;

(c) determine the coordinates of the center of mass of an asymmetrical body in two dimensions.

PROBLEMS

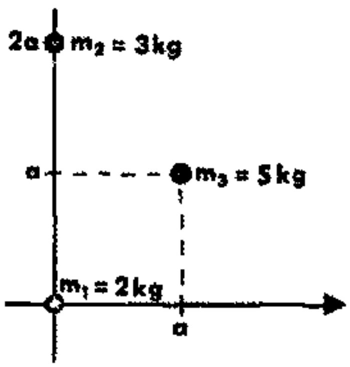
1. The coordinates of the center of mass of the system shown in the figure are



A. $x = a$; $y = 1.33\ a$
B. $x = 0.25\ a$; $y = a$
C. $x = a$; $y = 0.75\ a$
D. $x = 0.75\ a$; $y = a$

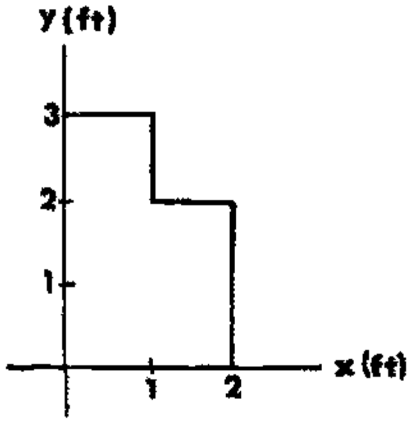
2. Consider the center of mass of a system of two particles m_1 and m_2 lying along the x-axis at x_1 and x_2, respectively. Write an expression for x_{cm}, the x-coordinate of the center of mass of the two particles.

3. What are the coordinates of the center of mass of the system shown in the figure?



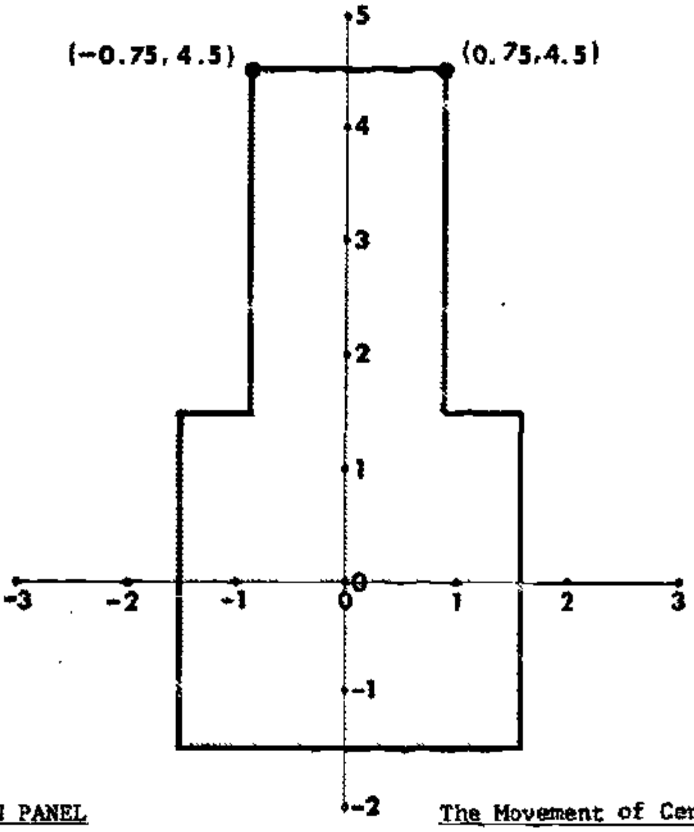
A. x = 2a; y = a
B. x = 0.5 a; y = 1.1 a
C. x = a; y = a
D. x = 0.33 a; y = a

4. A piece of 3/4 inch plywood has been cut into the shape shown. If uniform mass density and thickness are assumed for this piece of wood, then the center of mass is located at the point



A. (0.9, 1.0)
B. (1.3, 1.3)
C. (0.9, 1.3)
D. (1.0, 1.3)

5. A piece of 1/2 inch plywood has been cut into the shape shown. If uniform mass density and thickness are assumed for this piece of wood, what are the coordinates of its center of mass?



INFORMATION PANEL — The Movement of Center of Mass

OBJECTIVE

To correctly analyze problems involving the movement of the center of mass of a system of particles subjected to internal and/or external forces.

Since the basic definition of center of mass commits us to thinking of it as a point where all the mass of a body may be considered to be concentrated, it follows from Newton's second law that

> *The center of mass of a system of particles is a point which moves as though the total mass of the system is concentrated at that point and is subject to a force equal to the resultant of all external forces on the system.*

next page

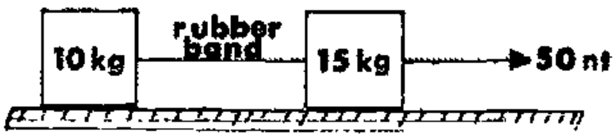
continued

Suppose a perfectly symmetrical hollow ball containing a compressed spring is made up of two segments fitted together. Suppose further that the ball is dropped vertically downward from the top of a bridge and that the spring is timed to decompress after the ball has fallen part of the way down, blowing the segments apart. The force exerted by the spring on the segments is an *internal* one; it must be accompanied by a reaction force which makes the net force on the segment system zero. All internal forces have this characteristic, hence the emphasis on *external* forces in the italicized statement above. Thus, internal forces cannot affect the motion of the center of mass of any system of moving particles. In this example, the two fragments of the ball would follow trajectories such that the center of mass of the two-fragment system would continue to *fall straight down as though the ball had not blown apart at all*. Note that no mention was made in the description about the relative masses of the two fragments. The apparent disregard of the center of mass for internal forces applies equally well to a pair of fragments having a mass ratio of, say, 10 to 1 as it does to a pair of fragments of equal mass. The only external force acting on the system--whether intact or segmented--is the downward force of gravitation so that the center of mass will have an acceleration equal to g just as the whole ball would have had if it had not come apart.

It is left as a thought exercise for you to visualize the difference in flight pattern of two equal-mass fragments as compared with two fragments of unequal mass.

All of the problems in this set require that you be able to recognize the differences in effect of internal and external forces acting on a system of particles.

6. Two masses on a table are connected by a rubber band. A constant force of 50 nt is applied to the right mass as shown. The coefficient of kinetic friction between each mass and the table is $\mu = 0.2$. The left mass is 10 kg and the right mass is 15 kg. What is the acceleration of the center of mass when both masses are moving to the right?



7. A weightlifter's barbell is accidentally released from the cargo hatch of a moving airplane. One of the weights separates from the bar in midair. Which statement best describes the resulting motion?

A. The center of mass of the whole barbell system will follow the same trajectory as the one it would follow if the weights did not separate.

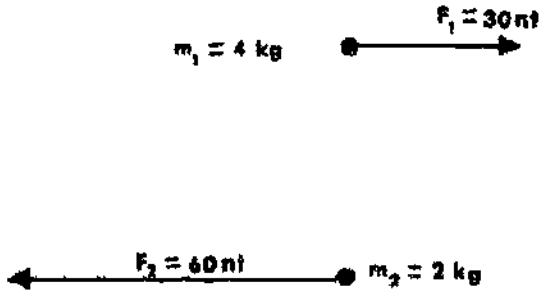
B. The motion of the pieces is completely random.

C. The pieces follow a path such that their center of mass falls to the ground along a straight vertical line.

D. The motion cannot be described because there is insufficient data.

8. A shell explodes in mid-trajectory near the surface of the Earth. Neglecting friction, name the geometric curve which describes the trajectory that the center of mass of the exploded shell will follow while both fragments are in flight.

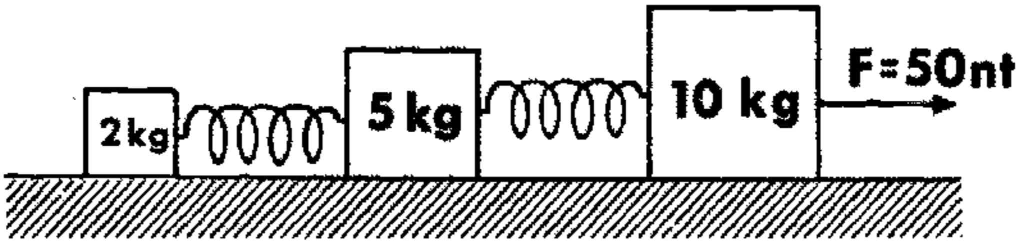
9.



For the system of masses and forces shown above, the acceleration of the center of mass is

A. 22.5 m/sec^2 towards left
B. 37.5 m/sec^2 towards right
C. 5 m/sec^2 towards left
D. 15 m/sec^2 towards right

10. Three masses on a table are connected by springs as shown in the figure below. A constant force of 50 nt is applied to the extreme right mass as shown. The coefficient of kinetic friction between each mass and the table is m = 0.2. The masses of the blocks are 2 kg, 5 kg and 10 kg as shown in the diagram. What is the acceleration of center of mass when all the masses are moving to the right?



[a] CORRECT ANSWER: Parabola

In this situation, the net external force on all the particles is equal to the weight of the whole system. Therefore, the center of mass of the fragmented shell moves as though it were the center of mass of the *intact* shell. Its trajectory will be parabolic because the internal explosive force cannot affect the motion of the center of mass.

[b] CORRECT ANSWER: 4 cm/sec^2

This problem looks much more difficult than it is. The acceleration of the center of mass is just the net external force divided by the total mass. The total mass is 25 kg. The net external force is the applied external force of 50 nt minus the frictional force exerted by the table. The total frictional force is

$$f = \mu N = \mu mg = \mu g(m_1 + m_2) = 0.2 \times 9.8 \times 25 = 49 \text{ nt}$$

Hence, the net external force is 50 - 49 = 1 nt, from which we obtain an acceleration

$$a = \frac{1}{25}\,\frac{\text{nt}}{\text{kg}} = 0.04 \text{ m/sec}^2 = 4 \text{ cm/sec}^2$$

The answer is independent of the speed of either mass (both moving to the right). If either mass were stationary or moving in the opposite direction, the frictional forces would be different from those calculated above. For moving blocks, we have to take into account that the frictional force is always directed opposite to the velocity. If a block is stationary, we must use μ_s instead of μ_k in our calculations.

Note that the forces exerted by the rubber band on the masses are internal forces. They must be taken into account when free-body diagrams are drawn for the two masses, but they do not affect the motion of the center of mass.

TRUE OR FALSE? The rubber band may be replaced by a massless string without changing the acceleration of the center of mass.

[a] CORRECT ANSWER: B

By definition

$$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i} \quad \text{and} \quad y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

Therefore

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2)(0) + (3)(0) + (5)(a)}{2 + 3 + 5} = 0.5a$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2)(0) + (3)(2a) + (5)(a)}{2 + 3 + 5} = 1.1a$$

Notice that although m_1 does not contribute to $\Sigma\, m_i\vec{r}_i$ (since $\vec{r}_1 = 0$), it must be included in $\Sigma\, m_i$.

TRUE OR FALSE? If m_1 is made 1 kg instead of 2 kg, it may then be omitted from the summations.

[b] CORRECT ANSWER: A

Before the pieces separated there were internal forces exerted by the pieces on each other. After the separation there is no interaction between the separate pieces. However, as far as Newton's second law is concerned, the situation has not changed. The forces before separation were internal, action-reaction forces and by Newton's third law their resultant was zero. Therefore, the net external force on the whole system has not changed with the separation.

[a] CORRECT ANSWER: (0.0, 1.0)

Divide the given piece of wood into two pieces: one square of area 9 ft^2 and another rectangle of area 4.5 ft^2. The centers of mass of these pieces are located at the points c_1 = (0,0) for the square and c_2 = (0,3) for the rectangle. Since the plywood is of uniform thickness, the masses of these two pieces are $m_1 = 9\rho$ and $m_2 = 4.5\rho$ where ρ is the assumed mass per ft^2 and m_1 and m_2 are the masses of the square and the rectangle, respectively. The total mass of the given piece is $M = 13.5\rho$. The problem now has been reduced to finding the center of mass of two particles of mass m_1 and m_2 located at c_1 and c_2 respectively.

$$x_{cm} = \frac{(9\rho \times 0.0 + 4.5\rho \times 0.0)}{13.5\rho} = 0.0 \text{ ft}$$

$$y_{cm} = \frac{(9\rho \times 0.0 + 4.5\rho \times 3)}{13.5\rho} = 1.0 \text{ ft}$$

TRUE OR FALSE? In a problem of this type, the position of the center of mass can be described only by giving at least three coordinates.

[b] CORRECT ANSWER: $(m_1x_1 + m_2x_2)/(m_1 + m_2)$

The mass-weighted mean of the positions of n particles is the center of mass of the system. The x-coordinate of the center of mass x_{cm} is given by

$$x_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

Thus, for the two particles in this question

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

[a] CORRECT ANSWER: D

By definition,

$$x_{cm} = \frac{\sum_i m_i x_i}{\sum_i m_i} \quad \text{and} \quad y_{cm} = \frac{\sum_i m_i y_i}{\sum_i m_i}.$$

So,

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(2)(0) + (2)(a) + (4)(a)}{2 + 2 + 4} = 0.75\ a$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(2)(0) + (2)(0) + (4)(2a)}{8} = a$$

Notice that, although m_1 does not contribute to $\Sigma\, m_i\vec{r}_i$ (since $\vec{r}_1 = 0$), it must be included in $\Sigma\, m_i$.

TRUE OR FALSE? One of the 2-kg masses has a y-coordinate equal to a.

[b] CORRECT ANSWER: 0.98 m/sec^2

The acceleration of the center of mass is the net external force divided by the total mass. The total mass is 17 kg. The net external force is the applied external force of 50 nt minus the frictional force exerted by the table. The total frictional force is

$$F = \mu N = \mu g(m_1 + m_2 + m_3)$$

$$= 0.2 \times 9.8\ \text{m/sec}^2 \times 17\ \text{kg}$$

$$= 33.32\ \text{nt}$$

Therefore, the acceleration is

$$a_{cm} = \frac{50\ \text{nt} - 33.32\ \text{nt}}{17\ \text{kg}} = 0.98\ \text{m/sec}^2$$

TRUE OR FALSE? In this problem, the friction due to air resistance has been ignored.

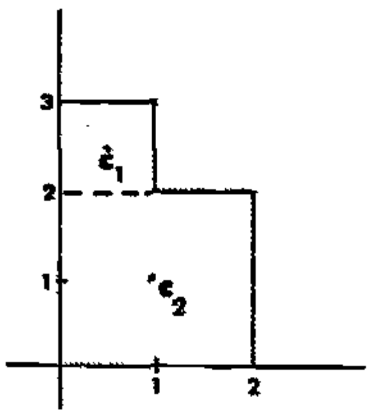
[a] CORRECT ANSWER: C

The center of mass of a system moves in the same way that a single particle of equal mass subject to the same external force would move. The resultant external force is F = 30 nt to the left and the total mass is $m_1 + m_2 = 6$ kg. Therefore, the acceleration of the center of mass is

$$\vec{a}_{cm} = \frac{\vec{F}}{m} = \frac{-30\ \hat{i}}{6}\ \frac{nt}{kg} = -5\ \hat{i}\ nt/kg = 5\ m/sec^2 \text{ toward left}$$

[b] CORRECT ANSWER: C

We shall present two ways of solving this problem. One way is to divide the given piece of wood into two squares of area 1 ft^2 and 4 ft^2, respectively. The centers of mass of these square pieces are located at the points $C_1 = (0.5, 2.5)$ and $C_2 = (1,1)$. Since the plywood is of uniform density and thickness, the masses of these two square pieces are respectively $m_1 = 1\rho$ and $m_2 = 4\rho$, where ρ is the assumed mass-per-unit-area. The total mass of the given piece is $M = 5\rho$. The problem now has been reduced to finding the center of mass of two particles of mass m_1 and m_2 located at C_1 and C_2, respectively.



$$x_{cm} = \frac{1}{5\rho}(\rho \times 0.5 + 4\rho \times 1) = \frac{0.5 + 4}{5} = 0.9 \text{ ft}$$

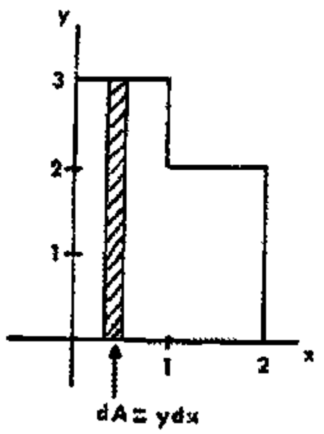
and

$$y_{cm} = \frac{1}{5\rho}(\rho \times 2.5 + 4\rho \times 1) = \frac{2.5 + 4}{5} = 1.3 \text{ ft}$$

next page

continued

Another way to do this problem is to view this board as a continuous object and use the equivalent integral expressions for locating the center of mass. Thus,



$$x_{cm} = \frac{1}{M}\int x\ dm = \frac{\rho}{M}\int x\ dA = \frac{\rho}{M}\int xy\ dx$$

The value of y is:

$y = 3$ for $0 \leq x \leq 1$, and

$y = 2$ for $1 \leq x \leq 2$

Therefore,

$$x_{cm} = \frac{\rho}{M}\left[3\int_0^1 x\ dx + 2\int_1^2 x\ dx\right] = \frac{\rho}{5\rho}\left[\frac{3}{2}\left(x^2\right)\Big|_0^1 + \frac{2}{2}\left(x^2\right)\Big|_1^2\right]$$

$$= \frac{1}{5}\left|\ 1.5 \times (1 - 0) + (4 - 1)\right| = \frac{1.5 + 3}{5} = 0.9 \text{ ft}$$

In a similar fashion

$$y_{cm} = \frac{1}{M}\int y\ dm = \frac{\rho}{M}\int yx\ dy$$

with:

$x = 2$ for $0 \leq y \leq 2$ and

$x = 1$ for $2 \leq y \leq 3$

Therefore,

$$y_{cm} = \frac{\rho}{5\rho}\left[2\int_0^2 y\ dy + \int_2^3 y\ dy\right] = \frac{1}{5}\left[\frac{2}{2}\left(y^2\right)\Big|_0^2 + \frac{1}{2}\left(y^2\right)\Big|_2^3\right]$$

$$= \frac{1}{5}\left[(4 - 0) + 0.5(9 - 4)\right] = \frac{4 + 2.5}{5} = 1.3 \text{ ft}$$

The latter method is, of course, more tedious. If, however, the piece of wood could not be divided into simple symmetrical parts (consider a semi-circle for example), the integral method would be the only one that could be used.

TRUE OR FALSE? The position of C_1 and C_2 in each square of wood in the first solution may be determined by symmetry considerations.

UNITED STATES NAVAL ACADEMY

Self-Paced Physics

SEGMENT SEPARATOR

note

ALL WRITTEN MATERIAL APPLICABLE TO THE FOLLOWING SEGMENT IS CONTAINED IN THE PAGES BETWEEN THIS COLORED SHEET AND THE NEXT.

INFORMATION PANEL The Momentum of a Particle

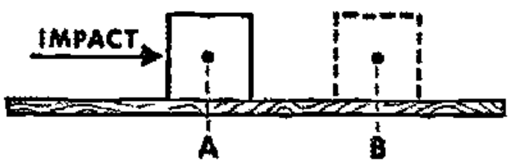
OBJECTIVE

To state and interpret the definition of momentum; to solve descriptive and numerical problems involving the momentum of particles with constant mass.

Although the word "momentum" appeared in the literature of classical physics subsequent to Newton's statement of his laws of motion, it is evident from his writings that he recognized the product of mass and velocity as an important physical entity. In his statement of the second law, he refers to what we now call momentum as the "quantity of motion" of a body. Since the word "quantity" is broad in meaning, the phrase "quantity of motion" is vague and unsatisfying.

Let's try to clarify its significance with the help of a simple example.

A steel block rests on a horizontal wood plank with its center of mass directly above point A. An experimenter, for reasons known only to himself, wishes to have the block moved to position B by permitting some other object moving toward the block from the left to collide with it. He has available to him only two things he can use: a hammer with a very massive head and a number of different cartridges together with the rifle that can fire them. He decides to strike the block with the hammer and, after a number of trials, find the *speed of impact* required with that particular hammer *mass* to successfully move the block of steel from A to B. In the second part of his experiment, he tries various bullet sizes and speeds until he finds one combination, say a .22 caliber slug with a definite muzzle velocity, which produces exactly the same motion of the block when allowed to strike it horizontally from the left. He now concludes that the hammer and the bullet had the same *quantity of motion* because they both produced the same motional change in the steel block. Thus, despite the crudity of the experiment and the possible hidden variables that may be present, he has explicitly defined quantity of motion in terms of the effect it has on another object.



next page

continued

It is clear that both speed and mass must enter into a judgment of the quantity of motion possessed by the moving body. From the preceding example, it can be shown that quantity of motion, and the product of mass and velocity are equivalent. Using p to represent quantity of motion or *momentum*:

$$p_{hammer} = Mv \qquad \text{and} \qquad p_{bullet} = mV$$

in which we have used upper and lower case letters to symbolize large and small quantities, respectively.

Thus, *momentum is an entity* comprising a mass term and a velocity term in the form of a product. Two moving bodies (i.e., a massive hammer and a bullet of small mass) can be given the same momentum despite the mass disparity by giving them the correct velocities.

Since mass is a scalar and velocity is a vector quantity, the product is a vector, hence the definition of momentum should be written:

$$\vec{p} = m\vec{v}$$

In the British system, the unit for momentum is the slug-ft/sec. In the MKS system it is the kg-m/sec. In problem work, caution in unit conversions must be exercised to be sure that the dimensions of momentum are properly expressed.

All of the statements regarding the momentum of a particle apply equally well to real bodies when all the mass of the body is considered to be concentrated at the center of mass.

When solving the problems in this set, you will be asked to

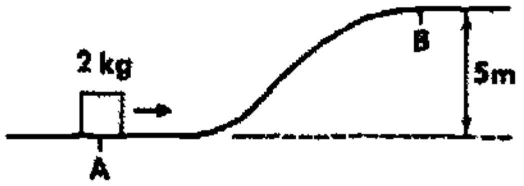
(a) determine the momentum of an object with given mass. In this problem, you will want to find the velocity of the body by applying the given data in the law of conservation of energy;

(b) solve a simple momentum problem involving unit conversions;

(c) determine momentum by first finding the velocity of a body from a knowledge of its kinetic energy.

PROBLEMS

1. A 2-kg block slides along the frictionless track shown in the figure. If the block's speed at point A is 10 m/sec, what is the momentum in kg-m/sec of the block at point B?

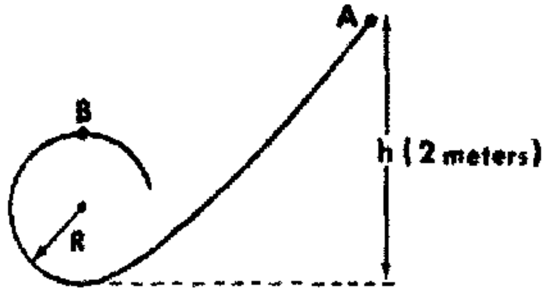


2. A 3200-lb automobile is heading north at a speed of 50 ft/sec. Its momentum is a vector directed north with a magnitude of

A. 160,000 lb-ft/sec
B. 160,000 slug-ft/sec
C. 5,000 slug-ft/sec
D. 5,000 lb-ft/sec

3. A 2-kg block slides with constant velocity down an inclined plane. The kinetic energy of the block is 16 joules. What is the magnitude of the block's momentum in kg-m/sec?

4. A particle of mass m = 2 kg slides down a track to enter an inside loop of radius R = 50 cm shown in the figure below. Without losing contact with the track at any time, it starts from rest at point A. What is its momentum at point B. Neglect friction.



OBJECTIVE

To extend momentum concepts to the solution of problems involving systems of particles.

A careful analysis of the behavior of a system of particles shows that

> *the total momentum of a system of particles may be found by multiplying the total mass of the system by the velocity of the center of mass of the system.*

or

$$\vec{P} = M\vec{v}_{cm}$$

in which $\vec{P}$ = the total momentum of the system,
M = the total mass of the system,
$\vec{v}_{cm}$ = the velocity of the center of mass.

This problem section involves:

(a) a determination of the velocity of the center of mass of a system given the momentum of the system;

(b) finding the resultant momentum of a pair of particles by the method of vector addition;

(c) a determination of the direction of the net momentum of a given system of particles using vector methods.

5. Two particles of mass 2 kg and 3 kg respectively, are moving with a speed of 10 m/sec due east. A third particle of mass 2 kg is moving with a speed of 25 m/sec due north. Determine the velocity of the center of mass, $\vec{v}_{cm}$, of the system of three particles.

A. 10.1 m/sec at 45° N of E
B. 20.2 m/sec at 37° N of E
C. 10.1 m/sec at 37° N of E
D. 20.2 m/sec at 45° N of E

6. Two bodies in a system have masses 8 kg and 12 kg and are moving with velocities 10 m/sec at 60° north of east and 5 m/sec at 30° south of east, respectively. The magnitude of momentum of the system is

A. 100 kg-m/sec
B. 140 kg-m/sec
C. 20 kg-m/sec
D. 70 kg-m/sec

7. A 2-kg particle moves due north at a speed of 1 m/sec. A second particle of mass 10 kg moves due east at a speed of 2 m/sec. What is the direction of the total momentum of the system?

A. 18° north of east
B. 12° north of east
C. 8° north of east
D. 6° north of east

8. A system of particles with masses of 8 kg and 12 kg has a total momentum of 100 kg-m/sec at 23° north of east. Determine the velocity of the center of mass of the system.

A. 100 m/sec; at 23° north of east
B. 140 m/sec; at due north
C. 20 m/sec; at 53° north of east
D. 5 m/sec; at 23° north of east

9. Two particles of mass $m_1 = 2$ kg and $m_2 = 3$ kg are moving with velocities of 10 m/sec due east and 20 m/sec due west, respectively. Determine the velocity of the center of mass, $\vec{v}_{cm}$, of the system.

A. 8 m/sec; due west
B. 16 m/sec; due east
C. 16 m/sec; due west
D. 8 m/sec; due north

INFORMATION PANEL — The Second Law in Terms of Momentum

OBJECTIVE

To utilize Newton's second law expressed in momentum terms in interpreting certain physical situations and solving problems related to these situations.

Emphasis has been placed previously on the constraint that mass must be constant if the second law in the form

$$F = ma$$

is to be valid. In a number of cases, the mass of the system continually varies so that this equation can no longer be applied. For example, as a chemically propelled rocket moves, it burns fuel continuously so that its mass correspondingly decreases with time. Such problems are most easily handled by applying momentum considerations and, for this reason, it is important for you to be able to apply the second law in momentum terms with facility.

Newton's expression of the second law in Latin, when translated freely in modern terminology reads

> *The rate at which the momentum of a body changes is proportional to the resultant force acting on the body and takes place in the direction of the straight line in which the force acts.*

or

$$\vec{F} = d\vec{p}/dt$$

next page

continued

If the mass is constant, the acceleration form of the second law is valid since

$$\vec{F} = d\vec{p}/dt = d(m\vec{v})/dt = m\, d\vec{v}/dt = m\vec{a}$$

You will find these formulations of the second law helpful in attacking the problems in this section.

10. The total mass of a system is 3 kg and the magnitude of the system's momentum is changing at the rate of 15 kg-m/sec^2. What is the magnitude of the net external force exerted on the system?

11. The total mass of a system is 100 gm, and the magnitude of the system's momentum is changing at the rate of 1000 gm-cm/sec^2. The magnitude of the acceleration of the center of mass of the system is

A. 1000 cm/sec^2
B. 10 cm/sec^2
C. 100,000 cm/sec^2
D. 98,000 cm/sec^2

12. The total mass of a system is 15 kg and the magnitude of the acceleration of its center of mass is 10 m/sec^2. What is the rate of change of the system's momentum?

INFORMATION PANEL Conservation of Momentum

OBJECTIVE

To apply the principle of conservation of momentum to the solution of typical problems in which this principle is found.

In the previous section of this segment of your work, you made use of Newton's second law in momentum terms, that is

$$\vec{F} = d\vec{p}/dt$$

in which $\vec{F}$ = the resultant force acting on the system, and $d\vec{p}/dt$ is the rate of change of momentum.

It follows directly from this that if the resultant force on the system is zero, then the rate of change of momentum of the system must also be zero, which in turn indicates that the momentum must remain constant.

$$\text{If } d\vec{P}/dt = 0\text{, then } \vec{P} = \text{constant.}$$

In verbal form, this conclusion may be stated as follows:

> *In any system of interacting particles, the total vector momentum remains constant unless the system is acted on by an external net force.*

This statement implies that, although the momenta of individual particles may change from one moment to the next, their vector sum remains the same as long as no resultant force is applied.

Since momentum is a vector quantity, you must expect to use vector methods in summing up the momenta of particle systems, or in resolving a given particle momentum into components. The problems in this set entail your recognition of the fact that when the net external force is zero, momentum is conserved.

13. An 8-ton, open-top freight car is coasting at a speed of 5 ft/sec along a frictionless horizontal track. It suddenly begins to rain hard, the raindrops falling vertically with respect to ground. Assuming the car to be deep enough, so that the water does not spatter over the top of the car, what is the speed of the car after it has collected 4.5 tons of water?

14. A midshipman dives from the stern of a stationary rowboat. His mass is 70 kg and that of the rowboat 140 kg. The horizontal component of his velocity when his feet leave the boat is 3 m/sec relative to the water. What is the speed of the boat immediately after the dive?

15. A block of wood of mass $M = 0.8$ kg is suspended by a cord of negligible mass. A bullet of mass $m = 4$ gm is fired horizontally at the block with a muzzle velocity of 400 m/sec. The bullet remains embedded in the block. What is the speed with which the wood block (with bullet embedded) is set into motion?

16. Assume a rocket has an initial weight of 3000 tons and a weight of 2780 tons after the fuel is completely burned. Fuel is consumed at a rate of 2840 lb/sec. After what time interval in seconds does the rocket attain its maximum velocity?

17. Let $\vec{v}$ be the velocity of a rocket (mass m) relative to ground and $\vec{u}$ the velocity of the exhaust gases relative to the rocket. Newton's second law then becomes

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} - \vec{u}\frac{dm}{dt}$$

To which of the following does this equation reduce if the rocket is being held stationary on a test pad by bolts, which exert a force $\vec{F}_b$ on the rocket?

A. $m\vec{g} - \vec{F}_b = -\vec{u}\frac{dm}{dt}$

B. $\vec{F}_b = -\vec{u}\frac{dm}{dt}$

C. $m\vec{g} + \vec{F}_b = -\vec{u}\frac{dm}{dt}$

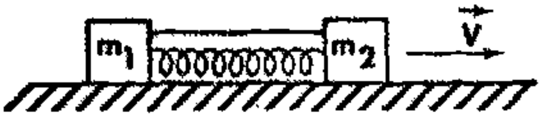
D. $m\vec{g} = -\vec{u}\frac{dm}{dt}$

18. Integrating the rocket equation given in problem 17 yields

$$\vec{v} = \vec{v}_o - \vec{u}\ln\left[\frac{m_o}{m}\right] + \vec{g}t$$

For the rocket and data given in Problem 16, determine the maximum velocity of the rocket if the exhaust velocity of the gases relative to the rocket is 55,000 m/sec and the rocket started from rest.

19. Two masses are tied together with a compressed spring between the two as shown. The spring is not attached to either mass. The system slides on a frictionless table with a velocity $\vec{v}$. At some point the string is cut and the masses fly apart along the original line of motion. The velocities of mass m_1 and m_2 after release are $\vec{v}_1$ and $\vec{v}_2$, respectively. What was the impulse imparted to mass m_2?



A. $m_2(\vec{v}_2 - \vec{v}_1)$

B. $m_1(\vec{v}_1 - \vec{v}_2)$

C. $m_2(\vec{v}_2 - \vec{v})$

D. $m_2(\vec{v}_1 - \vec{v})$

20. For the system in problem 19, what is the impulse imparted to m_1?

A. $-m_1(\vec{v}_1 - \vec{v})$

B. $-m_2(\vec{v}_2 - \vec{v})$

C. $m_1(\vec{v} - \vec{v}_1)$

D. $m_1(\vec{v}_2 - \vec{v})$

21. For the system in problem 19, what is the momentum of the center of mass of the system after the string has been cut and the masses have attained their final velocities?

A. $m_2(\vec{v}_2 - \vec{v})$

B. $m_1(\vec{v}_1 - \vec{v})$

C. $(m_1 + m_2)\dfrac{\vec{v}_1 + \vec{v}_2}{2}$

D. $(m_1 + m_2)\vec{v}$

[a] CORRECT ANSWER: $2\sqrt{2}$ kg-m/sec

Since non-conservative (frictional) forces are absent, the total energy of the block is conserved. If the potential energy is taken to be zero at point A, we have

$$K_A = K_B + mgh \qquad (1)$$

with the subscripts A and B referring to points A and B, respectively. Since

$$K = \frac{mv^2}{2}$$

(1) becomes

$$\frac{mv_A{}^2}{2} = \frac{mv_B{}^2}{2} + mgh$$

Solving for v_B, we have

$$v_B = \sqrt{v_A{}^2 - 2gh}$$

and the momentum p_B at point B is given by

$$p_B = mv_B = m\sqrt{v_A{}^2 - 2gh}$$
$$= 2\sqrt{100 - 98} = 2\sqrt{2}\ \text{kg-m/sec}$$

TRUE OR FALSE? The momentum of the block is independent of its position along the incline.

[b] CORRECT ANSWER: 15 nt

Newton's second law of motion can be expressed as

$$\vec{F} = \frac{d\vec{p}}{dt}$$

This shows that the force exerted on a body is equal to the time rate of change of its momentum. Hence, the magnitude of the force exerted on the given system is 15 kg-m/sec^2 = 15 nt.

TRUE OR FALSE? In this problem, $d\vec{p}/dt$ is equal to 15 kg-m/sec^2.

[a] CORRECT ANSWER: D

The total momentum $\vec{p}$ of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass,

$$\vec{p} = M\vec{v}_{cm}$$

Solving for $\vec{v}_{cm}$, we obtain

$$\vec{v}_{cm} = \frac{\vec{p}}{M} = \frac{100\ \text{kg-m/sec}}{(8 + 12)\ \text{kg}}\ (23^\circ\ \text{N of E}) = 5\ \text{m/sec}\ (23^\circ\ \text{N of E})$$

[b] CORRECT ANSWER: 150 kg-m/sec^2

Newton's second law of motion can be expressed as

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{F} = \text{net external force} \tag{1}$$

This shows that the external force exerted on a system is equal to the time rate of change of its momentum. However, the net external force exerted on a system is also given by

$$\vec{F} = M\vec{a}_{cm} \tag{2}$$

Where M is the total mass of the system and a_{cm} is the acceleration of its center of mass.

Equating equations (1) and (2) yields

$$\frac{dp}{dt} = M\vec{a}_{cm} = \text{the magnitude of the rate of change of momentum}$$

Substituting numerical data, we obtain

$$\frac{dp}{dt} = 15\ \text{kg} \times 10\ \text{m/sec}^2 = 150\ \text{kg-m/sec}^2$$

TRUE OR FALSE? The rate of change of the momentum of a system on which a net force acts is numerically equal to the rate of change of this force.

(a) CORRECT ANSWER: 1.99 m/sec

If v is the speed of the bullet and V the speed of the block with the bullet embedded in it, then by the conservation of momentum, we have

$$mv = (m + M)\,V \tag{1}$$

Therefore, the required speed V is

$$V = \frac{m}{m + M}\,v \tag{2}$$

Substitution of numerical values in equation (2) yields

$$V = \frac{4 \times 10^{-3}\ \text{kg}}{0.804\ \text{kg}} \times 400\ \text{m/sec}$$

$$= 1.99\ \text{m/sec}$$

TRUE OR FALSE? The magnitude of the force of gravity acting on the block and bullet is an important consideration in this solution.

(b) CORRECT ANSWER: 3.2 ft/sec

There are no external forces in the horizontal direction acting on the car-water system. Therefore, momentum is conserved. Thus,

$$m_i v_i = m_f v_f$$

and

$$v_f = \frac{m_i}{m_f}\,v_i = \frac{m_i g}{m_f g}\,v_i = \frac{8\ \text{tons}}{12.5\ \text{tons}} \times 5\ \text{ft/sec} = 3.2\ \text{ft/sec}$$

Note that, since the mass (or weight) of the system is involved in a ratio, no conversion to slugs (lb) is necessary.

TRUE OR FALSE? This problem could have been solved correctly by converting the weight of the car and the weight of the collected water to pounds before substituting the numbers.

[a] CORRECT ANSWER: C

The magnitude of the momentum is equal to the mass of the automobile times its speed. Since $w = mg$, $m = w/g$ and

$$mv = \frac{w}{g} v = \frac{3200 \text{ lb}}{32 \text{ ft/sec}^2} \times 50 \frac{\text{ft}}{\text{sec}} = 5000 \text{ slug-ft/sec}$$

[b] CORRECT ANSWER: 8.9 kg-m/sec

Since the frictional force is negligible, the system under consideration is conservative, and consequently the total energy of the point mass remains constant. If the potential energy is taken to be zero at the bottom of the loop, we have

$$mgh = mg(2R) + K_B \qquad (1)$$

Where K_B is the kinetic energy of the particle at point B. However,

$$K_B = \frac{P_B{}^2}{2m} \qquad P_B = \text{momentum of the particle at point B}$$

Therefore, equation (1) becomes

$$mgh = mg(2R) + \frac{P_B{}^2}{2m} \qquad (2)$$

Solving for P_B yields

$$\begin{aligned} P_B &= \sqrt{2m^2 g(h - 2R)} \\ &= \sqrt{2 \times 4 \times 9.8 \times 1} \\ &= 8.9 \text{ kg-m/sec} \end{aligned}$$

TRUE OR FALSE? The momentum of the sliding particle is greater at point B than it was at point A.

[a] CORRECT ANSWER: A

The momentum $\vec{P}$ of the center of mass is equal to the sum of the individual momenta. The resultant momentum in the easterly direction has a magnitude given by

$$P_E = (2\ \text{kg})(10\ \text{m/sec}) + (3\ \text{kg})(10\ \text{m/sec})$$

$$= 50\ \text{kg-m/sec}$$

In the northerly direction, the momentum has a magnitude P_N:

$$P_N = (2\ \text{kg})(25\ \text{m/sec}) = 50\ \text{kg-m/sec}$$

From the vector diagram of $\vec{P}_E$ and $\vec{P}_N$, we can calculate total momentum $\vec{P}$,

$$P = \sqrt{P_E^2 + P_N^2} = 71\ \text{kg-m/sec}$$

$$\theta = \tan^{-1}(P_N/P_E) = \tan^{-1}1 = 45°$$

Finally, $\vec{V}_{cm} = \vec{P}/M$

$$V_{cm} = P/M = 71/(2 + 3 + 2) = 10.1\ \text{m/sec}$$

TRUE OR FALSE? The velocity of the center of mass of this system is determined by adding the individual particle velocities algebraically.

[a] CORRECT ANSWER: A

The total momentum of a system of particles is equal to the product of the total mass of the system and the velocity of the center of mass,

$$\vec{P} = (m_1 + m_2)\, v_{cm} \qquad (1)$$

Where

$$\vec{P} = \vec{P}_1 + \vec{P}_2 = m_1\vec{v}_1 + m_2\, \vec{v}_2 \qquad (2)$$

Where $\vec{v}_1$, $\vec{P}_1$, and $\vec{v}_2$, $\vec{P}_2$ are the velocities and momenta of the particles of mass m_1 and m_2, respectively. Let the easterly direction lie along the positive x-axis so that the westerly direction will lie along the negative x-axis. Hence $\vec{v}_1$ and $\vec{v}_2$ have components in the x-direction only. Therefore,

$$\begin{aligned} P &= m_1v_1 - m_2v_2 \\ &= 2\text{ kg} \times 10\text{ m/sec} - 3\text{ kg} \times 20\text{ m/sec} \\ &= -40\text{ kg-m/sec} \end{aligned}$$

The magnitude of the total momentum is 40 kg-m/sec and is due west.

From equation (1) we have

$$\vec{v}_{cm} = \frac{\vec{P}}{(m_1 + m_2)}$$

Substitution of the numerical values yields

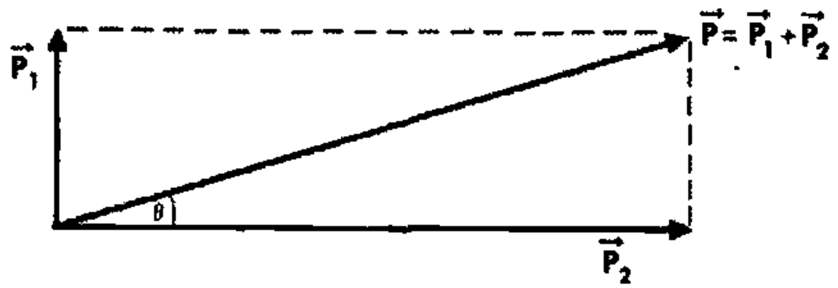
$$v_{cm} = \frac{40\text{ kg-m/sec}}{(2\text{ kg} + 3\text{ kg})} = 8\text{ m/sec}$$

The direction of v_{cm} is the same as that of the total momentum P, i.e., along the negative x-axis.

TRUE OR FALSE? The direction of the center-of-mass velocity vector is the same as that of the total momentum vector.

[a] CORRECT ANSWER: D

Introduce the notation $\vec{P}_1$ and $\vec{P}_2$ for the momenta of the 2-kg and 10-kg particles, respectively. We want to calculate the angle θ as shown in the diagram below.



From the geometry, we have

$$\tan\theta = P_1/P_2$$

substitute values of P_1 and P_2 to find

$$\tan\theta = (2\text{ kg} \times 1\text{ m/sec})/(10\text{ kg} \times 2\text{ m/sec})$$

$$= 0.1$$

Therefore,

$$\theta = \tan^{-1}0.1 \approx 6°$$

[a] CORRECT ANSWER: C

The external forces on the rocket are those exerted by gravity and the bolts, both downward. Since the rocket is stationary, dv/dt is zero, and the "rocket equation",

$$\vec{F}_{ext} = m\frac{d\vec{v}}{dt} - \vec{u}\frac{dm}{dt}$$

reduces to

$$\vec{F}_{ext} = -\vec{u}\frac{dm}{dt}$$

Finally, the external force is given by

$$\vec{F}_{ext} = m\vec{g} + \vec{F}_b = -\vec{u}\frac{dm}{dt}$$

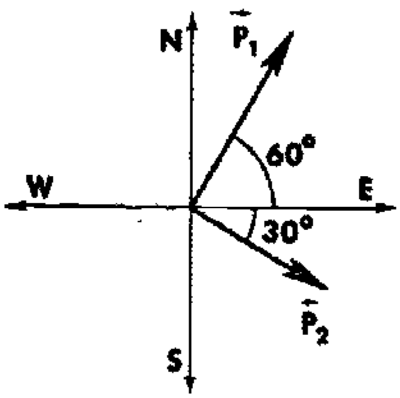
[b] CORRECT ANSWER: D

The whole system was moving with a velocity $\vec{v}$ before the string was cut. Hence, $\vec{v}$ is the initial velocity of the center of mass. Since the table is frictionless, the external force acting on the system, both before and after the string is cut, is zero. Therefore, the momentum (and velocity) of the system remain unchanged. The final velocity of the center of mass is $\vec{v}$ and the momentum is $(m_1 + m_2)\,\vec{v}$.

TRUE OR FALSE? The momentum of the system remains unchanged because all of the forces involved in this interaction are internal ones.

[a] CORRECT ANSWER: A

The momentum of a system is the vector sum of the individual momenta.



$\vec{p}_1$ = 8 kg × 10 m/sec = 80 kg-m/sec; at 60° N of E

$\vec{p}_2$ = 12 kg × 5 m/sec = 60 kg-m/sec; at 30° S of E

Using the fact that $\vec{p}_1$ and $\vec{p}_2$ form a right angle, we find

$$p = \sqrt{p_1^2 + p_2^2} = \sqrt{80^2 + 60^2} = 100 \text{ kg-m/sec}$$

[b] CORRECT ANSWER: B

For a system whose mass is constant we have

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

so

$$a = \frac{1}{m}\frac{dp}{dt} = \frac{1000 \text{ gm-cm/sec}^2}{100 \text{ gm}} = 10 \text{ cm/sec}^2$$

[a] CORRECT ANSWER: C

From the impulse-momentum principle we know that to find the impulse imparted to m_2, we must compute the change in the momentum of m_2. Thus,

$$\vec{J}_2 = \Delta\vec{p}_2 = \vec{p}_{2f} + \vec{p}_{2i} = m_2(\vec{v}_2 - \vec{v})$$

TRUE OR FALSE? Frictional forces must be considered when calculating the change in momentum of mass 2.

[b] CORRECT ANSWER: 155 seconds

The maximum velocity will occur at the instant the fuel has been consumed. Until then, the rocket accelerates; thereafter, it decelerates.

The rocket is losing mass (or weight) at a constant rate. Thus

$$\frac{d(mg)}{dt} = \frac{dw}{dt} = -2840\ \text{lb/sec}$$

or

$$\int_0^t dt = -\frac{1\ \text{sec}}{2840\ \text{lb}} \int_{w_i}^{w_f} dw$$

and

$$t = -\frac{1\ \text{sec}}{2840\ \text{lb}}(w_f - w_i) = -\frac{1\ \text{sec}}{2840\ \text{lb}}\left[(2780 - 3000)\ \text{tons} \times 2000\ \frac{\text{lb}}{\text{ton}}\right]$$

$$= 155\ \text{sec}$$

TRUE OR FALSE? According to the conditions given in the solution, this rocket must be traveling horizontally in a vacuum.

[a] CORRECT ANSWER: 8 kg-m/sec

The known quantity is kinetic energy K,

$$K = mv^2/2$$

where m is the mass of the block, and v is its speed.

Solving for v,

$$v = \sqrt{2k/m}$$

so the momentum, p, is given by

$$p = mv = m\sqrt{2k/m}$$

$$= \sqrt{2mk}$$

$$= 8 \text{ kg-m/sec}$$

[b] CORRECT ANSWER: 1.5 m/sec

The momentum of the system is zero before the dive. In the absence of an external force, momentum is conserved during the dive; therefore, the momentum of the system after the dive is also zero. We simply have to solve the equation $m_1v_{1x} + m_2v_{2x} = 0$ for v_{2x}. Thus,

$$v_{2x} = -\frac{m_1v_{1x}}{m_2} = -\frac{70 \text{ kg} \times 3 \text{ m/sec}}{140 \text{ kg}} = -1.5 \text{ m/sec}$$

the minus sign indicating that $\vec{v}_{2x}$ is directed oppositely to $\vec{v}_{1x}$.

[a] CORRECT ANSWER: 2710 m/sec

With $v_0 = 0$ and the upward as the positive direction in the given equation we obtain

$$v = u \ln\left[\frac{m_0}{m}\right] - gt$$

The maximum velocity is attained at $t = 155$ sec, at which time $mg = 2780$ tons. Now $m_0 g = 3000$ tons and $u = 55{,}000$ m/sec, so

$$v_{max} = 55{,}000 \ln\left[\frac{3000}{2780}\right] - [9.8 \times 155] = 2710 \text{ m/sec}$$

TRUE OR FALSE? The weight of the rocket reaches its minimum value at the instant that the rocket attains its maximum speed.

[b] CORRECT ANSWER: B

The impulse imparted to m_1 is

$$\vec{J}_1 = \Delta\vec{p} = \vec{p}_{1f} - \vec{p}_{1i} = m_1(\vec{v}_{1f} - \vec{v}_{1i}) = m_1(\vec{v}_1 - \vec{v}) \qquad (1)$$

This expression, however, is not listed among the answer choices.

We may use Newton's third law of motion which states that the force exerted by m_1 on m_2 (with the spring as intermediary), is equal to the negative of the force exerted by m_2 on m_1. Since the forces also act for the same duration we have

$$\vec{J}_1 = \int_{t_i}^{t_f} \vec{F}_{12}\, dt = -\int_{t_i}^{t_f} \vec{F}_{21}\, dt = -\vec{J}_2$$

Using the result of the preceding problem, we obtain

$$\vec{J}_1 = -\vec{J}_2 = -m_2(\vec{v}_2 - \vec{v})$$

Self-Paced

PHYSICS

STUDY GUIDES

NEW YORK INSTITUTE of TECHNOLOGY

P	STEP	NAME
	0.1	Reading: *HR 4-4/4-6; 6-3; 11-5 SZ 6-6 AB 9-5; 9-6
	0.2	Information Panel, "The Vocabulary of Circular Motion"
1		573 rev/min (13a) — T: X, F: ✓ _______ (ans)
	1.1	Information Panel, "The Characteristics of Uniform Circular Motion"
2		A: X, B: X, C: 18a, D: X — T: ✓, F: X
	2.1	If your first choice was correct, advance to 7.1; if not, continue sequence.
3		T: X, F: direction (11a) _______ (ans)
4		A: X, B: X, C: X, D: 13c
5		A: X, B: 13b, C: X, D: X
6		A: 11c, B: X, C: X, D: X
7		A: X, B: 14b, C: X, D: X — T: X, F: ✓
	7.1	Audiovisual. CIRCULAR MOTION
8		v_1=0.01256 m/sec(19a) v_1 = _______ (ans) v_2=0.314 m/sec (19a) — T: ✓, F: X v_2 = _______ (ans)

P	STEP	SECTION SEGMENT 6
	8.1	Information Panel, "Centripetal Force"
9		1.85 sec (12a) — T: ✓, F: X _______ (ans)
	9.1	If correct, advance to P 14; if not, continue sequence.
10		A: X, B: 17a, C: X, D: X
11		A: 11b, B: X
12		10 m/sec (15a) _______ (ans)
13		30° (20a) — T: ✓, F: X _______ (ans)
14		30 rev/min (16a) — T: X, F: ✓ _______ (ans)
15		A: X, B: 14a, C: X, D: X — T: ✓, F: X
16		23.8 m/sec (17b) — T: X, F: ✓ _______ (ans)
	16.1	Homework: HR Problem 4-20.

REVEALED

SECTION — SEGMENT 7

P	STEP	NAME	A	B	C	D	T	F
	0.1	Reading: *HR Chapter 7; SW 6-1; 6-7; SZ 7-1/7-3, 7-9; 7-10; AB 6-1/6-7; 27-1/27-3						
	0.2	Information Panel, "Work Done by a Constant Force"						
1			X	X	X	26b	✓	X
2		480 ft-lb (21b) _______ (ans)					X	✓
	2.1	If correct, advance to 4.1; if not, continue sequence.						
3			X	X	X	28b		
4		30° (20a) _______ (ans)					✓	X
	4.1	Audiovisual, WORK WHEN FORCE VARIES IN MAGNITUDE AND DIRECTION						
	4.2	Information Panel, "Work Done by a Varying Force"						
5			X	X	33a	X	✓	X
	5.1	If your first choice was correct, advance to 8.1; if not, continue sequence.						
6		10 nt (17a) _______ (ans)						
7		300 j (24a) _______ (ans)						
8			X	X	X	17c	X	✓
	8.1	Information Panel, "Algebraic Signs in Work Problems"						
9			X	X	31a	X	✓	X
	9.1	Information Panel, "Unit Systems for Work and Power"						
10			26a	X	X	X	X	✓
	10.1	If your first choice was correct, advance to 14.1; if not, continue sequence						
11			X	X	X	19a		
12		980 watts (31b) _______ (ans)						
13		75, 350 ft-lb/sec or 137 hp (35a) _______ (ans)						
14			X	28a	X	X	✓	X
	14.1	Information Panel, "Kinetic Energy"						
	14.2	Audiovisual, KINETIC ENERGY						
15		100 j (30a) _______ (ans)					X	✓
	15.1	If correct, advance to 17.1; if not, continue sequence.						

REVEALED

P	STEP	NAME	P	STEP	SECTION SEGMENT 7
16		3.6×10^5 ft-lb (23a) (ans)	25		100 ft-lb (27b) (ans)
17		10 m/sec (27a) T ✓ F X (ans)	26		5 cm (32a) T ✓ F X (ans)
	17.1	Information Panel, "The Work-Energy Theorem"	27		A X, B X, C X, D 29a; T ✓ F X
18		10 m/sec (32b) T ✓ F X (ans)		27.1	If your first choice was correct, advance to 29.1; if not, continue sequence.
	18.1	If correct, advance to 23.1; if not, continue sequence.	28		A X, B 34a, C X, D X
19		A X, B 21a, C X, D X	29		A 22a, B X, C X, D X; T X F ✓
20		A X, B X, C 25b, D X		29.1	Homework: HR Problem 7-8
21		A X, B X, C X, D 17b			
22		825 lb (36a) (ans)			
23		8 m/sec (25a) T X F ✓ (ans)			
	23.1	Information Panel, "Composite Problems Involving Work and Energy"			
24		128 m (18a) T ✓ F X (ans)			
	24.1	If correct, advance to P 27; if not, continue sequence.			

P	STEP	NAME	P	STEP	SECTION SEGMENT 8
14		0.78 i (13a) (ans)			
15		0.8 m (17b) (ans)			
16		A: X, B: 19a, C: X, D: X			
17		A: X, B: 15a, C: X, D: X; T: ✓, F: X			
	17.1	Information Panel, "A Composite Problem Using Conservation of Energy"			
18		25 ft (18a); T: ✓, F: X (ans)			
	18.1	Homework: HR 8-16			

REVEALED

U.S. NAVAL ACADEMY — STUDY GUIDE — SELF-PACED PHYSICS

P	STEP	NAME
	0.1	Reading: HR 9-1, 9-2; 8-9 (opt) SW 6-6; 9-3 SZ 3-5 AB 30-4, 30-5
	0.2	Information Panel, "Coordinates of the Center of Mass of a System of Particles"
1		A: X, B: X, C: X, D: 11a; T: X, F: ✓
	1.1	If your first choice was correct, advance to P 4; if not, continue sequence.
2		$(m_1x_1+m_2x_2)/(m_1+m_2)$ (10b) ________ (ans)
3		A: X, B: 9a, C: X, D: X; T: X, F: ✓
4		A: X, B: X, C: 12b, D: X; T: ✓, F: X
	4.1	If your first choice was correct, advance to 5.1; if not, continue sequence.
5		(0.0, 1.0) (10a); T: X, F: ✓ ________ (ans)
	5.1	Information Panel, "Movement of the Center of Mass"
	5.2	Audiovisual, MOVEMENT OF CENTER OF MASS
6		4 cm/sec^2 (8b); T: ✓, F: X ________ (ans)

P	STEP	SECTION — SEGMENT 9
	6.1	If your first choice was correct, advance to 10.1; if not, continue sequence.
7		A: 9b, B: X, C: X, D: X
8		Parabola (8a) ________ (ans)
9		A: X, B: X, C: 12a, D: X
10		0.98 m/sec^2 (11b); T: ✓, F: X ________ (ans)
	10.1	Homework: HR 9-4

REVEALED

U.S. NAVAL ACADEMY — **STUDY GUIDE** — **SELF-PACED PHYSICS**

SEGMENT 10

P	STEP	NAME / SECTION	A	B	C	D	T	F
	0.1	Reading: *HR 9-3/9-6; 10-1/10-3 AB 30-1/30-3						
	0.2	Information Panel, "The Momentum of a Particle"						
1		$2\sqrt{2}$ kg-m/sec (12a) ____ (ans)					X	✓
	1.1	If correct, advance to 4.1; if not, continue sequence.						
2			X	X	15a	X		
3		8 kg-m/sec (22a) ____ (ans)						
4		8.9 kg-m/sec (15b) ____ (ans)					✓	X
	4.1	Information Panel, "Momentum of a System of Particles"						
5			16a	X	X	X	X	✓
	5.1	If your first choice was correct, advance to 9.1; if not, continue sequence.						
6			20b	X	X	X		
7			X	X	X	18a		
8			X	X	X	13a		
9			17a	X	X	X	✓	X
	9.1	Information Panel, "The Second Law in Terms of Momentum"						
10		15 nt (12b) ____ (ans)					✓	X
	10.1	If correct, advance to 12.1; if not, continue sequence.						
11			X	20b	X	X		
12		150 kg-m/sec^2 (13b) ____ (ans)					X	✓
	12.1	Information Panel, "Conservation of Momentum"						
	12.2	Audiovisual, CONSERVATION OF MOMENTUM						
13		3.2 ft/sec (14b) ____ (ans)					✓	X
	13.1	If correct, advance to P 16; if not, continue sequence.						
14		1.5 m/sec (22b) ____ (ans)						
15		1.99 m/sec (14a) ____ (ans)					X	✓

REVEALED

P	STEP	NAME	P	STEP	SECTION SEGMENT 10
16		T F 155 sec (21b) X ✓ (ans)			
	16.1	If correct, advance to P 19; if not, continue sequence.			
17		A B C D X X 19a X			
18		T F 2710 m/sec (23a) ✓ X (ans)			
19		A B C D T F X X 21a X X ✓			
	19.1	If your first choice was correct, advance to 21.1; if not, continue sequence.			
20		A B C D X 23b X X			
21		A B C D T F X X X 19b ✓ X			
	21.1	Homework: HR 9-15			

Self-Paced

PHYSICS

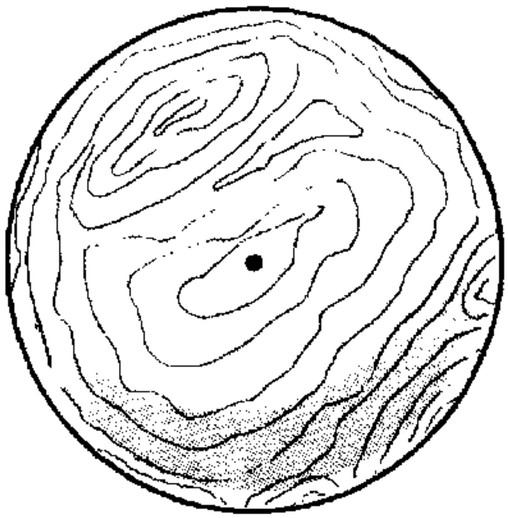
ILLUSTRATED TEXT

NEW YORK INSTITUTE of TECHNOLOGY

MOVEMENT OF CENTER OF MASS

CENTER OF MASS

(a) for a solid ball



(b) for a hollow ball

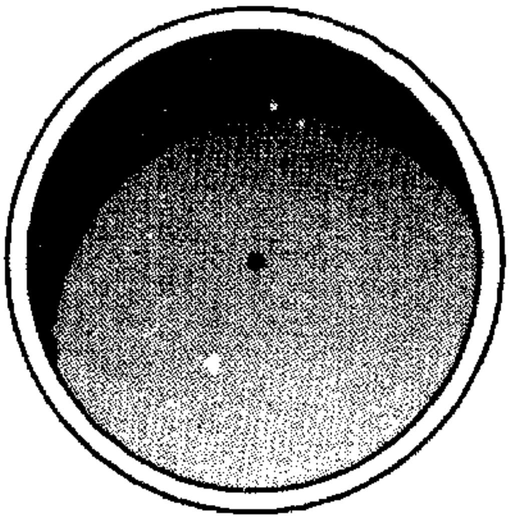


FIGURE 1

The center of mass of an object may be described as that single point at which all of its mass appears to act. For an object of uniform density having some regular shape, such as a solid wooden ball, its center of mass is easily located to be at the geometric center, as you can see in Figure 1. Finding the location of the center of mass for a hollow rubber ball is no more difficult--it too is at the geometric center, even though none of the actual mass of the ball is located at that very point.

Many objects, having either regular or irregular shapes, have centers of mass located in space--probably the chair you are sitting on at this moment or the cup or glass you used this morning are good examples to consider. For these objects, the center of mass acts in every way just as it does for one having a center of mass within the medium itself--as with the solid ball.

EQUAL MASS CARS

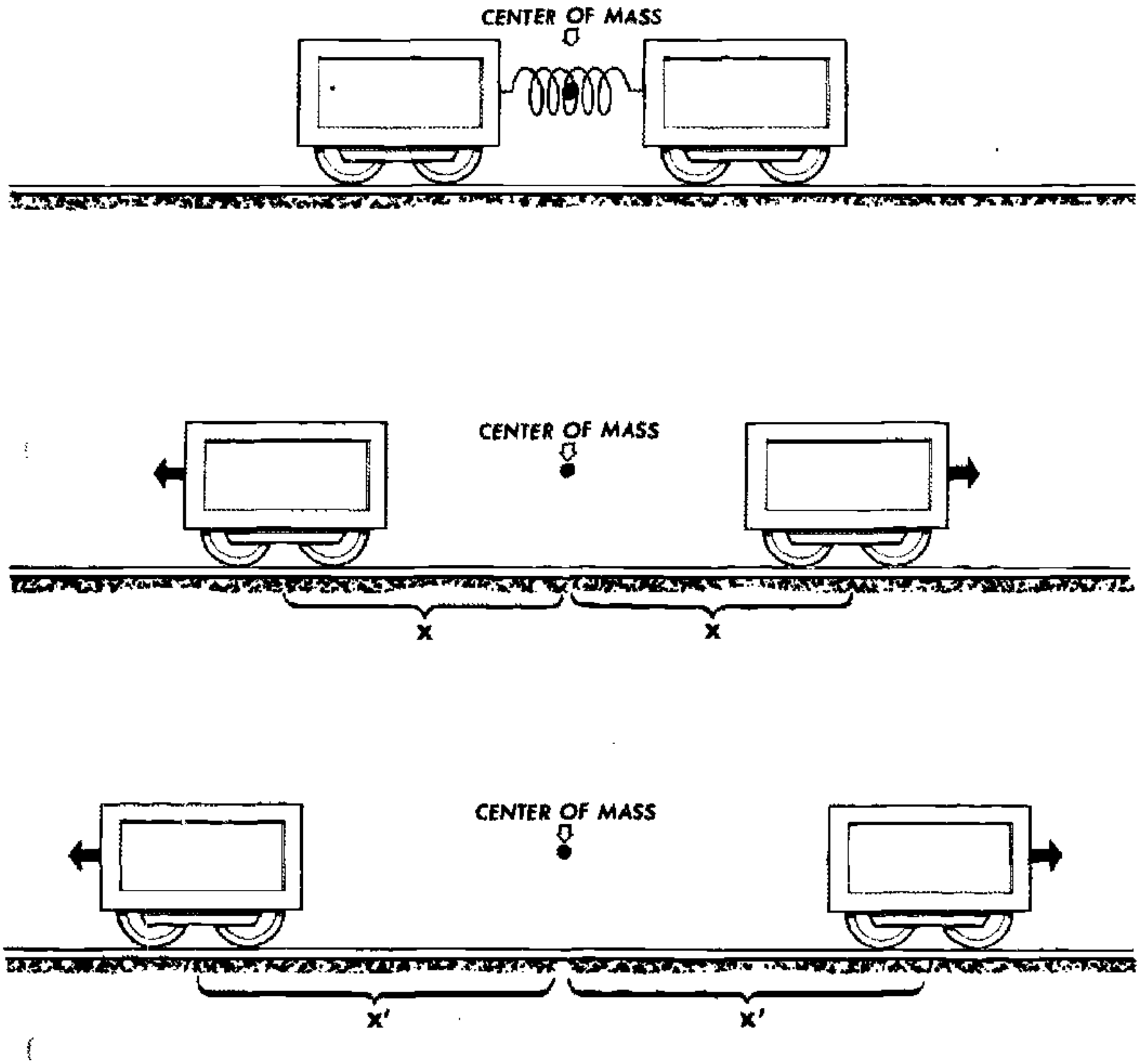


FIGURE 2

The concept of center of mass can be a powerful tool in the study of motion, since all rigid bodies, regardless of shape, volume, or density, can be considered to be point masses acted upon by external forces, thereby simplifying the application of Newton's laws of motion.

A task that at first seems difficult is the analysis of the motion of a body when internal forces are also acting. Let's see what effect, if any, they might have. To do this, let's examine the effect of an explosion on the center of mass of a system consisting of two equal masses. In Figure 2, you see two identical cars about to be exploded apart by a compressed spring. Before the explosion, the center of mass of the system is midway between the cars. When the explosion occurs, each car receives an equal, but opposite force to the other, for the same period of time, giving each similar accelerations. But at any time, the center of mass of the system can be found to be at the same point, unaffected by the explosion.

UNEQUAL MASS CARS

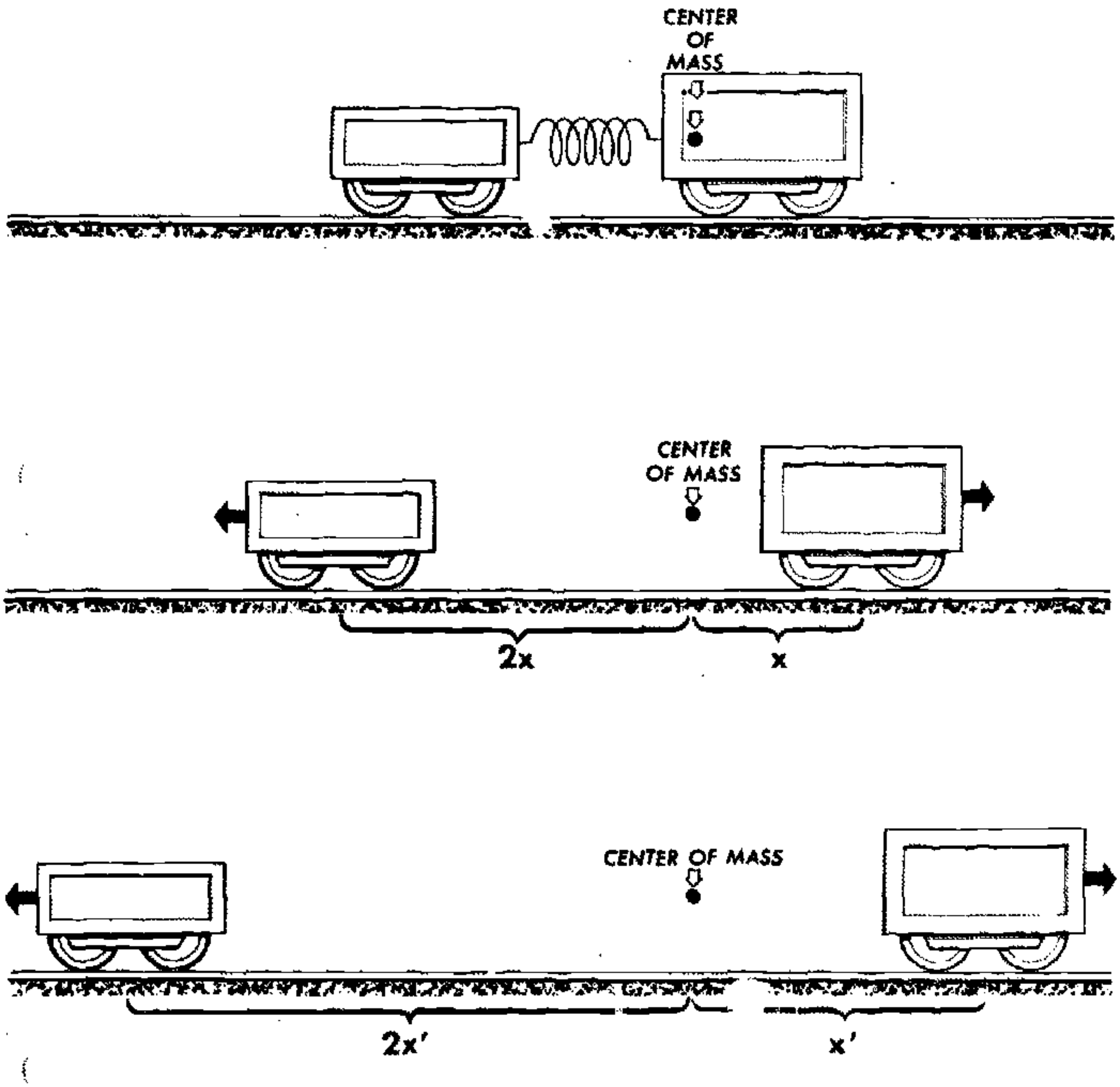


FIGURE (3)

You may well ask, what would have happened if two unequal masses were chosen? Let's repeat the explosion, this time with unequal cars; say they have a mass ratio between them of 1:2. Once again the explosion will apply equal and opposite forces on the cars, but this time one car, the lighter one, will accelerate at twice that of the heavy car, thereby moving twice as far in equal time. Consequently, the center of mass of the system remains in the same position, unaffected by internal forces as you can see by examining Figure 3. As a matter of fact, even if the two cars have some initial velocity while linked together, their center of mass would continue to move at that velocity even after the explosion occurs.

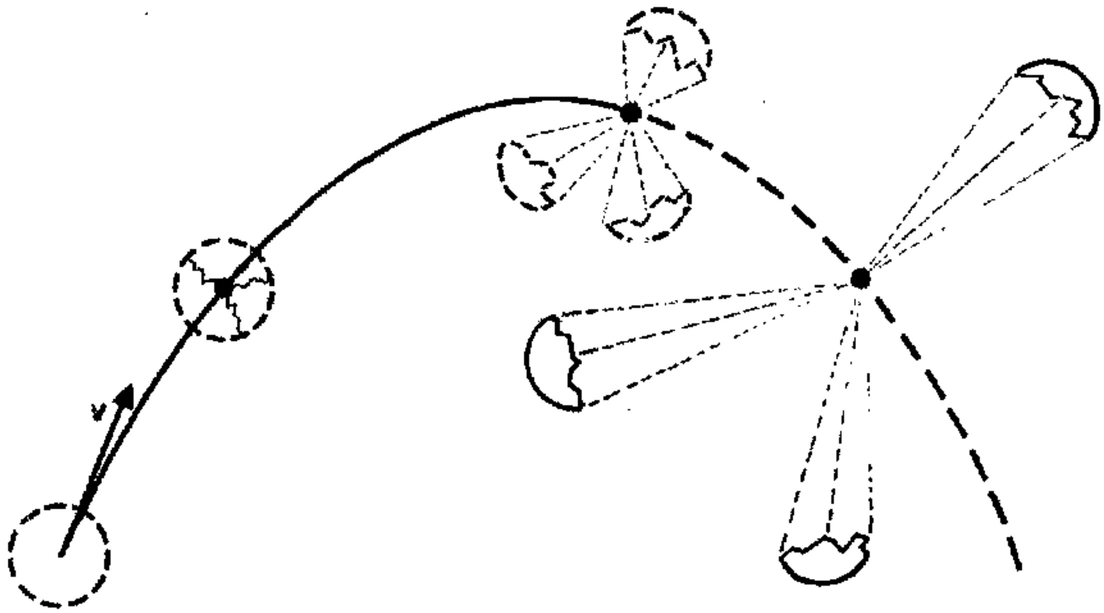


FIGURE 4

Before closing, let's apply these principles to some typical motion problem. A good one to consider would be the motion of an explodable ball as it moves in a parabolic trajectory. Here, in Figure 4, the ball is subjected to some initial accelerating force, and a constant gravitation force, both acting externally. as well as an internal explosive force.

Before the explos : the ball travels intact along a parabolic path governed by the effects of its initial velocity and gravitation. The ball is then exploded into fragments, each moving away from the center of gravity at a rate dependent upon the explosive force and its size. and each still is affected by the initial velocity and gravitation. Since the explosive internal force has been shown to have no effect on the center of gravity, its motion continues along the parabolic trajectory as though the ball had remained intact.

Self-Paced

PHYSICS

TALKING BOOK

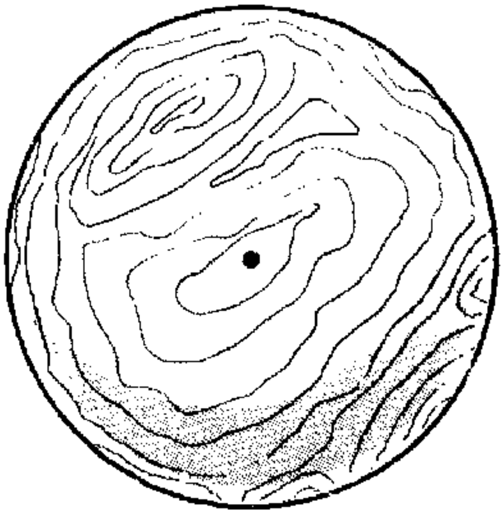
(WITH CASSETTE)

NEW YORK INSTITUTE of TECHNOLOGY

MOVEMENT OF CENTER OF MASS

CENTER OF MASS

(a) for a solid ball



(b) for a hollow ball

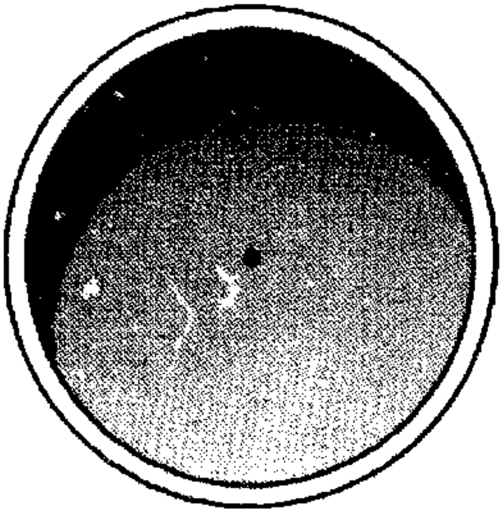


FIGURE 1

EQUAL MASS CARS

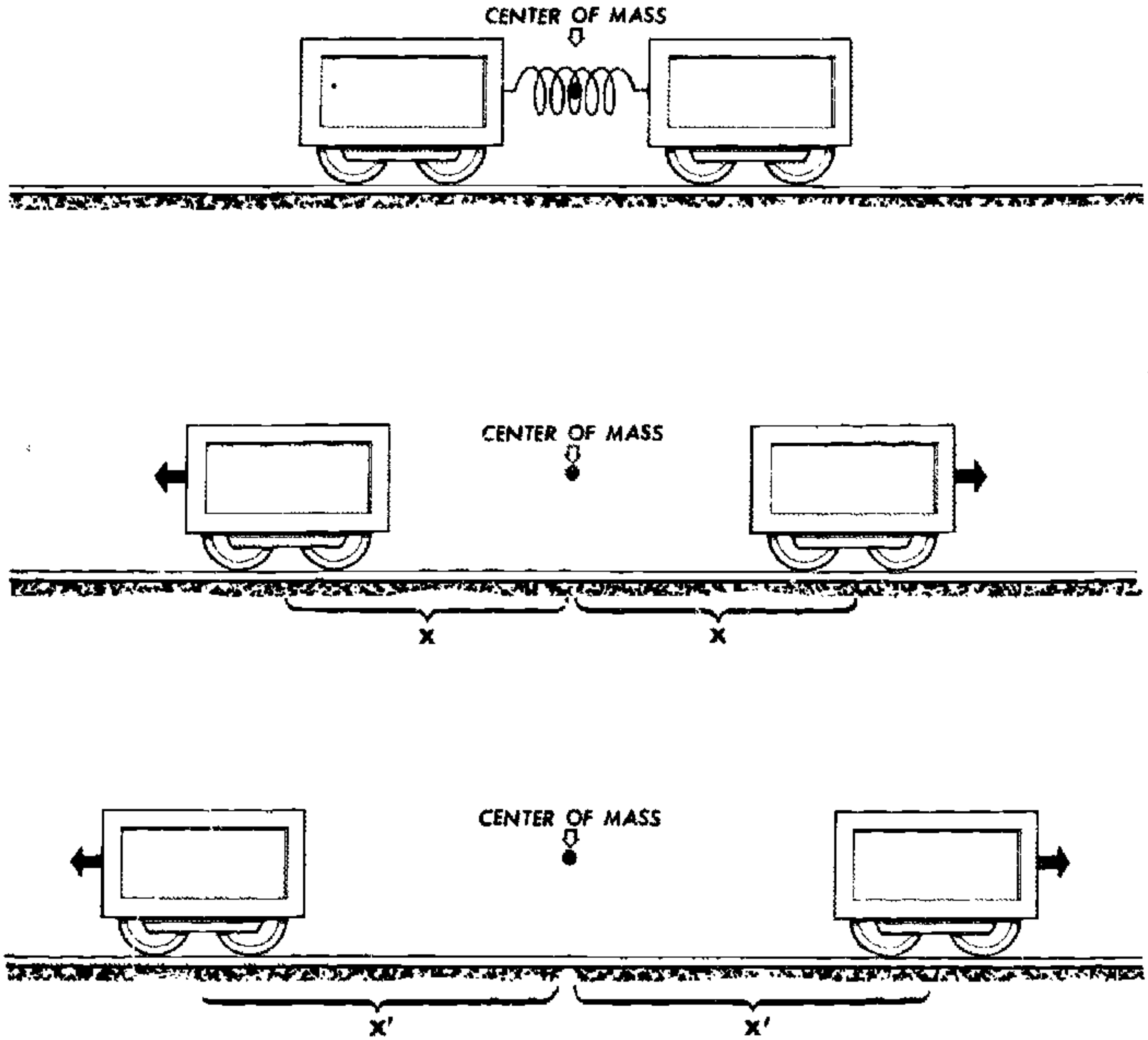


FIGURE 2

UNEQUAL MASS CARS

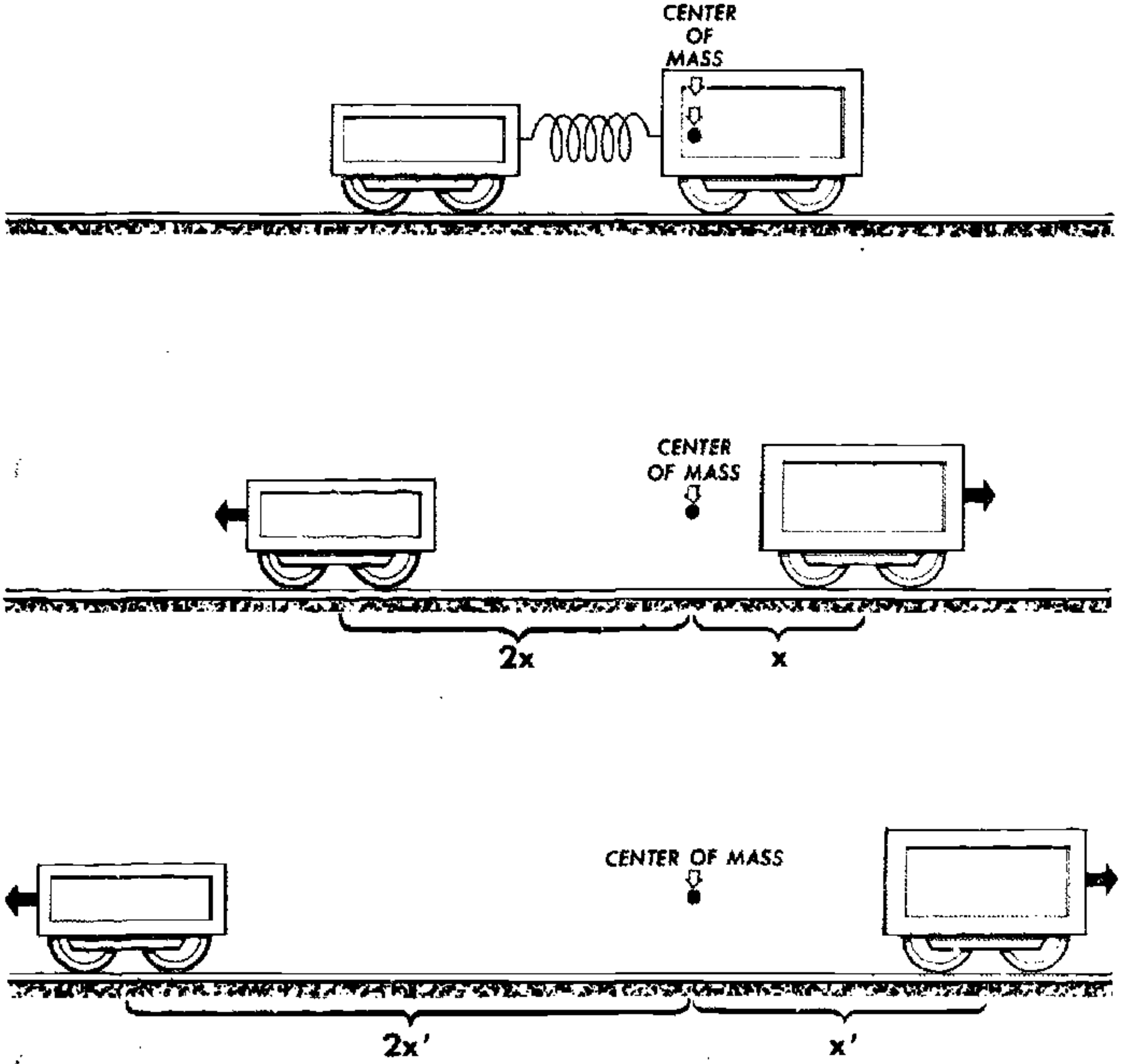


FIGURE 3

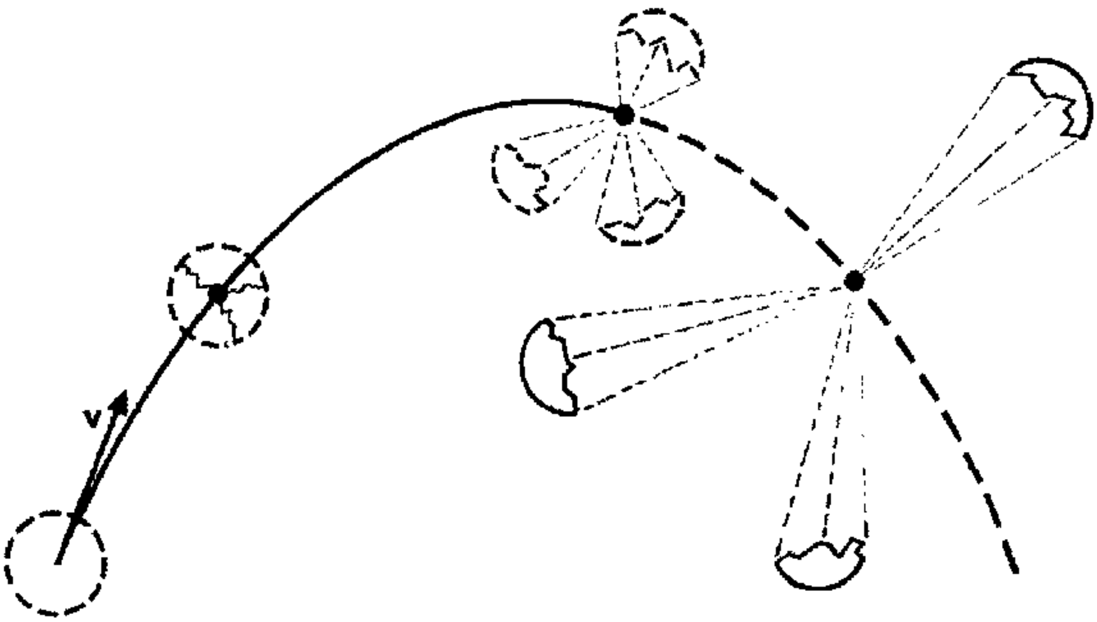


FIGURE 4

Self-Paced

PHYSICS

DIAGNOSTIC TEST

NEW YORK INSTITUTE of TECHNOLOGY

DIAGNOSTIC TEST - DELTA

T.O. 1 RR

A mile is approximately equivalent to:

(A) 1.6 km

(B) 0.6 km

(C) 0.45 km

(D) 2.54 km

T.O. 2 RR or CU

In the equation for constant velocity

$$v = \frac{(x - x_o)}{t}$$

(A) x and x_o depend upon the frame of reference and t does not depend upon the frame of reference

(B) x and x_o do not depend upon the frame of reference and t does depend upon the frame of reference

(C) x, x_o, and t depend upon the frame of reference

(D) x, x_o, and t do not depend upon the frame of reference

T.O. 3 CU

Express the sum of the numbers 15, 140.001, and 0.57

(A) 155.571

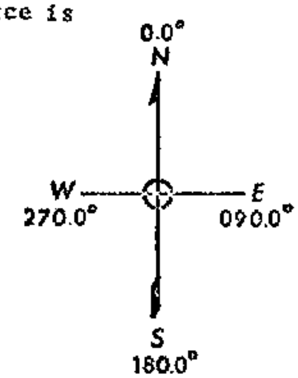
(B) 155.57

(C) 156

(D) 160

T.O. 4 CU

Two forces act simultaneously on the same point. Their values are 5.0 nt at 045.0° and 5.0 nt at 180.0°. The direction of the resultant force is



(A) Between 0° and 90°

(B) Between 90° and 180°

(C) Between 180° and 270°

(D) Between 270° and 360°

T.O. 5 RR/CU

The center of mass of a hollow sphere

(A) is located at the geometric center even though no mass is present at that location.

(B) is distributed th.oughout the mass since it cannot be located in empty space.

(C) does not exist at all for a sphere without mass at its center.

(D) forms its own spherical surface which touches everywhere the inside surface of the hollow sphere.

T.O. 6 PS

A boy throws a baseball vertically upward. If the ball is caught 4.0 seconds later, what height did it attain?

(A) 264 m

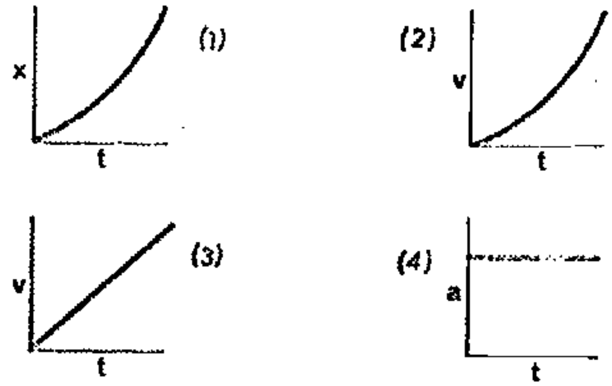
(B) 78 m

(C) 64 m

(D) 19.6 m

T.O. 7 CR

In which one of the following graphs can we be sure that the acceleration is varying?



(A) 1 (B) 2 (C) 3 (D) 4

T.O. 8 CU

In the equation

$$\alpha = v_o + \frac{1}{2} a(2t - 1 \text{ sec})$$

a = acceleration and t = time. From analysis of the dimensions, α is the equation of

(A) position

(B) speed

(C) acceleration

(D) has no meaning since it is dimensionally inconsistent

T.O. 9 CR

To test for the gravitational acceleration, g, a ball is dropped from rest from a height m and falls for the time t to the ground. The gravitational acceleration, g, can be found by:

(A) $g = \frac{2m}{t^2}$

(B) $g = \frac{4m^2}{t^2}$

(C) $g = \frac{2m}{t}$

(D) Insufficient data. Must know impact velocity to solve.

T.O. 10 PS/CA

A man walks toward the rear of a moving train while his motion is observed by a station attendant standing on a station platform. If the train moves to the right at 10 ft/sec relative to the stationery platform observer, while the walking man moves at 8 ft/sec to the right relative to the same station attendent; how fast does the man walk relative to the train?

(A) 18 ft/sec to the right

(B) 18 ft/sec to the left

(C) 2 ft/sec to the right

(D) 2 ft/sec to the left

T.O. 11 CU

The instantaneous velocity may be determined from
$v = at$ only for

(A) variable acceleration

(B) variable velocity

(C) constant acceleration

(D) constant velocity

T.O. 12 CR

A baseball player hits a fly ball whose trajectory reaches a maximum height of h; the time the outfielder has to position himself for his catch can be found by:

(A) $\frac{2h}{g}$

(B) $\frac{4h}{g}$

(C) $\sqrt{\frac{2h}{g}}$

(D) $2\sqrt{\frac{2h}{g}}$

T.O. 1[illegible] CR

How long does it take for a force F to change the speed of an object from v_o to v if its mass is m?

A. $t = \frac{m(v_o - v)}{F}$

B. $t = \frac{m(v - v_o)}{F}$

C. $t = \frac{(v - v_o)}{Fm}$

D. $t = \frac{(v_o - v)}{Fm}$

T.O. 15 CR

The weight of an astronaut (mass m) in orbit at an altitude above the Earth (mass M) equal to the Earth's radius, R, can be found from

A. $W = GMmR^2$

B. $W = 4GMmR^2$

C. $W = G\frac{Mm}{R^2}$

D. $W = G\frac{Mm}{4R^2}$

T.O. 16 CR

A man tries to push his stalled car on a level road. The maximum force he is able to apply is $\vec{F}$, but this is insufficient to move the car. The reaction to his force is [illegible]

[illegible]

(B) $-\vec{F}$

(C) $2\vec{F}$

(D) zero, since the car does not move

T.O. 17

When a block slides down a plane at uniform speed, the coefficient of kinetic friction is equal to

(A) the sine of the angle of inclination.

(B) the cosine of the angle of inclination.

(C) the tangent of the angle of inclination.

(D) a more complex function of the angle.

T.O. 18 CR

The period of each revolution, τ, of an object moving uniformly with a speed v in a circular path of radius r can be expressed as:

(A) $2\pi r/v$

(B) $v/2\pi r$

(C) $4\pi^2 r^2/v$

(D) $v/4\pi^2 r^2$

T.O. 19 CU

A coin of mass m is placed on a stationary phono turntable at a distance r from the spindle. The switch is turned on and the turntable begins to accelerate. If the coefficients of friction are respectively μ_s and μ_k (static and kinetic) the magnitude of the centripetal force F_c on the coin just before the coin starts to slide is

A. $F_c > \mu_s mg$

B. $F_c < \mu_s mg$

C. $F_c = \mu_s mg$

D. none of the above

A force stretches a spring with a spring c[illegible] an amount x from its equilibrium position (t[illegible] force and x are in the same direction). The work done by this force is

A. $+kx$

B. $+ 1/2\ kx^2$

C. $- 1/2\ kx^2$

D. $+ 1/2\ kx$

T.O. 21 CU

A woman begins to lift a pail of water out of a well; the initial total weight is W. The pail has a leak, however, and as the pail is lifted a distance y, water is slowly lost. The work of the woman is

A. Wy

B. $\frac{1}{2} Wy$

C. $\frac{1}{2} Wy^2$

D. unable to be determined from the information given

T.O. 22 RR

The power P developed by a machine which does an amount of work W in time t is

A. $P = Wt$

B. $P = Wt^2$

C. $P = W/t$

D. $P = W/t^2$

T.O. 23 RR

Which of the following statements is not true?

(A) One-half of the product of the mass of a body and the square of its speed is called the kinetic energy of the body.

(B) The work done by the resultant force acting on a body is equal to the change in the kinetic energy of the body.

(C) The kinetic energy of a body in motion is equal to the work it can do in being brought to rest.

(D) The kinetic energy is a function of position whose negative derivative gives the force.

T.O. 24 CU

Which of the following forces is *not* conservative?

(A) the frictional force

(B) the gravitational force

(C) the force exerted by an ideal spring

(D) the force exerted on a charge in an electric field

T.O. 25 RR

The statement of the conservation of mechanical energy is

(A) $\Delta K + \Delta U = 0$

(B) $W_{nc} = \Delta K$

(C) $W_{nc} = \Delta K + \Delta U$

(D) $\Delta U = 0$

where W_{nc} is the work done by nonconservative forces.

T.O. 26 [illegible]

A spring of constant k compressed a distance x [illegible] energy equal to

(A) mgx

(B) mkx

(C) 1/2 kx

(D) 1/2 kx^2

T.O. 27 CK

The mass of a simple pendulum bob is m. It is displaced slightly from its equilibrium position such that the bob is a height h above its equilibrium level. It is now released from rest. Its velocity at the bottom of its swing can be computed from

(A) mgh = 1/2 mv2

(B) gh = mv

(C) 1/2 gh^2 = 1/2 mv^2

(D) gh = $2v^2$

T.O. 28 CU

Which of the following is a correct statement regarding the center of mass of a circular ring?

(A) It is the entire outer surface of the ring.

(B) It cannot be the geometrical center of the ring because there is no material at this point.

(C) It may be exterior to the ring, depending upon the mass distribution of the ring.

(D) It is the geometrical center of the ring when the mass distribution is symmetrical around the center.

T.O. 29 CU

Two particles move toward each other. The center of mass of this system

(A) remains equidistant from each particle.

(B) becomes closer to the heavier particle and further from the lighter particle.

(C) becomes closer to the lighter particle and further from the heavier particle.

(D) becomes closer to both particles.

T.O. 30 CR

Two bodies each of mass 3 kg are moving eastward; one with a velocity of 2 m/sec, the other with a velocity of 4 m/sec. The magnitude of the total momentum of the system is

(A) 6 kg-m/sec

(B) 12 kg-m/sec

(C) 18 kg-m/sec

(D) 60 kg-m/sec

T.O. 33 CU

A ball strikes the floor, its initial velocity making an angle θ with the normal. It rebounds with the same speed also at an angle θ with normal. (The total angular change in direction of the ball is 180° - 2θ) What is the direction of the average impulsive force exerted on the ball by the floor?

(A) vertically upward

(B) vertically downward

(C) at an angle θ upward

(D) horizontally along the floor

Self-Paced

PHYSICS

COMPETENCE CHECK PROBLEMS

NEW YORK INSTITUTE of TECHNOLOGY

SAMPLE COMPETENCE CHECKS

SELF-PACED PHYSICS

4-2.2

A particle is set in motion along a horizontal frictionless surface at a speed of five feet per second. What is its speed at the end of seven seconds?

4-5.4

A man lowers vertically a 50-lb ball at a constant speed of 3 ft/sec. The magnitude of the force he applies to the ball

A. decreases as the ball descends.

B. increases as the ball descends.

C. is less than 50 lb.

D. is equal to 50 lb.

3-1.3

A sled moves from rest along a straight horizontal track with a constant acceleration of 10 ft/sec^2. At the end of ten seconds (10 sec) its engine cuts off and it comes to rest with a constant deceleration of 4 ft/sec^2. What is the total distance traveled by the sled?

A. 1,750 ft.

B. 875 ft.

C. 500 ft.

D. 1,000 ft.

E. 1,250 ft.

3-18.2

A projectile has an initial speed of 176 ft/sec. Assume that the projectile is initially at ground level, and that air resistance may be neglected. What is the maximum range of the projectile?

A. 242 ft.

B. 484 ft.

C. 726 ft.

D. 968 ft.

E. 1,210 ft.

4-11.4

Near the surface of Saturn, objects fall with an acceleration of 11.8 m/sec^2. What is the weight of a 4000 gram mass at Saturn's surface?

w = ______________

Self-Paced

PHYSICS

FOREWORD

AND

TWO SAMPLE CHAPTFRS

NEW YORK INSTITUTE of TECHNOLOGY

FOREWORD

Our new book has innovative pedagogical features, but it is designed so that the instructor may use it as he would any other introductory physics text. No revisions in syllabi or lectures are required--the topical coverage is a familiar one. Reading and homework problems can be assigned as usual. Of course, the instructor may greatly increase the effectiveness of this book through his active participation.

A feature of the text is an emphasis on problem solving. Much of the exposition is in the form of problem statements and their solutions. It is reasoned that since students are tested and evaluated by problems, problem-oriented instruction is most relevant for them. A consequence of this emphasis is that the lecturer may safely spend more time bringing physics to life for his students, and less time grinding through examples.

The course objectives are embodied in problem statements, the *core* problems. The importance of such goal-directing problems can hardly be over emphasized. If test questions are distributed well in advance of a conventional examination, test performance is predictably high. Similar high achievement can be expected if, in order to avoid outright memorization, the questions for advance distribution are known to be minor variations of the actual examination questions. When such a test and its precursor are expanded to cover the entire course content at appropriate levels, we then have a rough parallel to this goal-directed approach.

Core problems correspond to test questions distributed in advance.

Variations of these are called *core prime* problems and can be viewed as corresponding to terminal examination questions. Of course, a core prime problem is used as a self-test rather than an instrument for grading.

Each section of the text begins with a discussion of theory followed by the associated core problem statement. A student can attempt to solve the core problem and then choose one of three options on the basis of his Performance: proceed to the next section, attempt to solve the core prime problem, or read and work through a sequence of *enabling* problems which illustrate major steps in the solution of the core problem. The format for a section is:

Theory

Core Problem and Solution

Enabling Problem 1 and Solution

.

.

.

Enabling Problem N and Solution

Core Prime Problem

Thus, when a student is able to solve a core problem with confidence, he maximizes his progress by moving directly to the next section. Some students expect that they can execute problems similar to the core after having seen the correct solution. Often, such students have made a minor error in the core and only need a similar problem for practice and reinforcement; this is provided by the core prime problem. When a student incorrectly assesses his ability to solve the core prime, or when he realizes that his understanding of the core problem is deficient, then he takes the enabling problem sequence. The core prime problem is always encountered at the end of the enabling sequence.

The text should have special appeal for students because it teaches those facts and skills which are generally tested. When a student can solve the core problem or minor modification thereof, he has attained the learning objective. Unlike most conventional texts, the student knows exactly what is expected of him. Obviously, the instructor can contribute greatly to the intended plan simply by declaring that his test questions will be either variations of core problems or fragments of core problems.

It is not surprising that many professionals regard such goal-directed teaching as tantamount to cheating. They have tacitly accepted that an examination in basic physics really should test more than was taught; it should help determine scientific *aptitude*, *originality*, and *imagination*. This may serve a useful purpose, but until we know how to teach these qualities it seems reasonable to separate them from examinations purporting to measure gains in knowledge.

The principles and approach taken in the book have been found to increase performance on the objectives (core problems) by 57% over the traditional format of a theory section alone.

Each chapter concludes with a set of *review* problems. These are categorized as "A" and "B" problems corresponding respectively to simple exercises and core-level problems. The "Overview" serves as an example of interstitial material which will introduce all major topic areas (mechanics, thermodynamics, optics, wave motion, electromagnetism, and modern physics).

Finally, the book is ideally suited to self-study. Perhaps an unusual application of the book will be to abolish conventional class meetings and schedule hours during which the instructor is available for more individualized tutorial assistance.

CHAPTER 10

SAMPLE

Linear momentum is a physical quantity which, like mechanical energy, is conserved under certain general conditions. Much of the beauty and utility of such conserved quantities is a result of their relating "initial" and "final" events without consideration of any detailed intermediate processes.

In this chapter, our objectives are to define momentum both for individual particles and systems of particles, and to introduce the principle of conservation of momentum.

10-1 The Definition of Momentum

The linear momentum $\vec{p}$ of a particle is the product of the mass m and velocity $\vec{v}$:

$$\vec{p} = m\vec{v} \tag{10-1}$$

Notice that momentum is a vector quantity, so that in order to specify the momentum of a particle completely, the magnitude and direction must both be given.

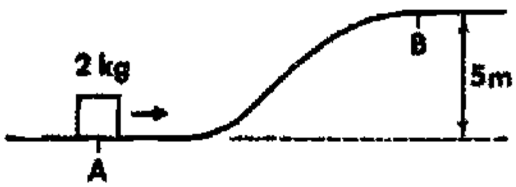
Linear momentum is so named to distinguish it from angular momentum, although most often, linear momentum is referred to simply as momentum. No special name is given to its unit; it is slug-ft/sec in the British system and kg-m in the MKS system.

The definition of momentum for a particle also applies to real bodies when all of the mass of the body is considered to be concentrated at the center of mass.

This problem section requires that you determine the magnitude of momentum for a body of given mass using energy considerations to find the velocity.

PROBLEM

A 2-kg block slides along the frictionless track shown in the figure. If the block's speed at point A is 10 m/sec, what is the momentum in kg-m/sec of the block at point B?



SOLUTION

Since non-conservative (frictional) forces are absent, the total energy of the block is conserved. If the potential energy is taken to be zero at Point A, we have

$$K_A = K_B + mgh \qquad (1)$$

with the subscripts A and B referring to points A and B, respectively. Since

$$K = \frac{mv^2}{2}$$

(1) becomes

$$\frac{mv_A^2}{2} = \frac{mv_B^2}{2} + mgh$$

Solving for v_B, we have

$$v_B = \sqrt{v_A^2 - 2gh}$$

and the momentum p_B at point B is given by

$$p_B = mv_B = m\sqrt{v_A^2 - 2gh}$$
$$= 2\sqrt{100 - 98} = 2\sqrt{2} \text{ kg-m/sec}$$

ENABLING PROBLEMS

1. A 3200-lb automobile is heading north at a speed of 50 ft/sec. Its momentum is a vector directed north with a magnitude of

A. 160,000 lb-ft/sec
B. 160,000 slug-ft/sec
C. 5,000 slug-ft/sec
D. 5,000 lb-ft/sec

SOLUTION

The magnitude of the momentum is equal to the mass of the automobile times its speed. Since $w = mg$, $m = w/g$ and

$$mv = \frac{w}{g} v = \frac{3200 \text{ lb}}{32 \text{ ft/sec}^2} \times 50 \frac{\text{ft}}{\text{sec}} = 5000 \text{ slug-ft/sec}$$

2. A 2-kg block slides with constant velocity down an inclined plane. The kinetic energy of the block is 16 joules. What is the magnitude of the block's momentum in kg-m/sec?

SOLUTION

The known quantity is kinetic energy K,

$$K = mv^2/2$$

where m is the mass of the block, and v is its speed.

Solving for v,

$$v = \sqrt{2k/m}$$

so the momentum, p, is given by

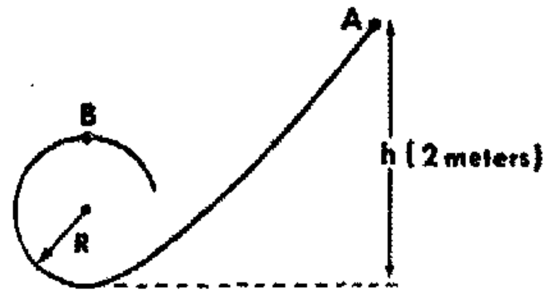
$$p = mv = m\sqrt{2k/m}$$

$$= \sqrt{2mk}$$

$$= 8 \text{ kg-m/sec}$$

COMPETENCE CHECK 10-1

A particle of mass $m = 2$ kg slides down a track to enter an inside loop of radius $R = 50$ cm shown in the figure below. Without losing contact with the track at any time, it starts from rest at point A. What is its momentum at point B. Neglect friction.



10-2 Momentum of a System of Particles

The total momentum $\vec{P}$ of a system of n particles is simply the vector sum of all the individual particle momenta:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \ . \ . \ . + \vec{p}_n \tag{10-2}$$

Total momentum is directly related to the velocity $\vec{v}_{cm}$ of the system's center of mass. To see this, we differentiate the center of mass coordinates

$$x_{cm} = \frac{\Sigma\ m_i\ x_i}{\Sigma\ m_i} \qquad y_{cm} = \frac{\Sigma\ m_i\ y_i}{\Sigma\ m_i} \qquad z_{cm} = \frac{\Sigma\ m_i\ z_i}{\Sigma\ m_i}$$

with respect to time. The result is

$$\vec{v}_{cm} = \frac{\Sigma\ m_i\ \vec{v}_i}{\Sigma\ m_i}$$

or, using the definition of momentum, this becomes

$$\vec{v}_{cm} = \frac{\Sigma\ \vec{p}_i}{\Sigma\ m_i}$$

The numerator of this expression is just the total momentum $\vec{P}$ and the denominator is the total mass of the system M. With these substitutions, we obtain the useful result

$$\vec{P} = M\vec{v}_{cm} \tag{10-3}$$

The problem section involves finding the total momentum and center of mass velocity for a pair of particles whose masses and velocities are known.

PROBLEM

Two particles of mass 2 kg and 3 kg respectively, are moving with a speed of 10 m/sec due east. A third particle of mass 2 kg is moving with a speed of 25 m/sec due north. Determine the velocity of the center of mass, $\vec{v}_{cm}$, of the system of three particles.

A. 10.1 m/sec at 45° N of E
B. 20.2 m/sec at 37° N of E
C. 10.1 m/sec at 37° N of E
D. 20.2 m/sec at 45° N of E

SOLUTION

The momentum $\vec{P}$ of the center of mass is equal to the sum of the individual momenta. The resultant momentum in the easterly direction has a magnitude given by

$$P_E = (2\text{ kg})(10\text{ m/sec}) + (3\text{ kg})(10\text{ m/sec})$$

$$= 50\text{ kg-m/sec}$$

In the northerly direction, the momentum has a magnitude P_N:

$$P_N = (2\text{ kg})(25\text{ m/sec}) = 50\text{ kg-m/sec}$$

From the vector diagram of $\vec{P}_E$ and $\vec{P}_N$, we can calculate total momentum $\vec{P}$,

$$P = \sqrt{P_E^2 + P_N^2} = 71\text{ kg-m/sec}$$

$$\theta = \tan^{-1}(P_N/P_E) = \tan^{-1}1 = 45°$$

Finally, $\vec{V}_{cm} = \vec{P}/M$

$$V_{cm} = P/M = 71/(2 + 3 + 2) = 10.1\text{ m/sec}$$

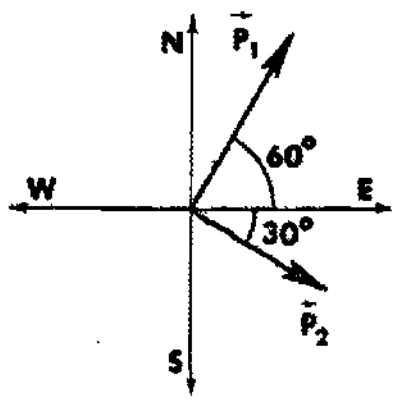
ENABLING PROBLEMS

1. Two bodies in a system have masses 8 kg and 12 kg and are moving with velocities 10 m/sec at 60° north of east and 5 m/sec at 30° south of east, respectively. The magnitude of momentum of the system is

A. 100 kg-m/sec
B. 140 kg-m/sec
C. 20 kg-m/sec
D. 70 kg-m/sec

SOLUTION

The momentum of a system is the vector sum of the individual momenta.



$\vec{p}_1 = 8$ kg × 10 m/sec = 80 kg-m/sec; at 60° N of E

$\vec{p}_2 = 12$ kg × 5 m/sec = 60 kg-m/sec; at 30° S of E

Using the fact that $\vec{p}_1$ and $\vec{p}_2$ form a right angle, we find

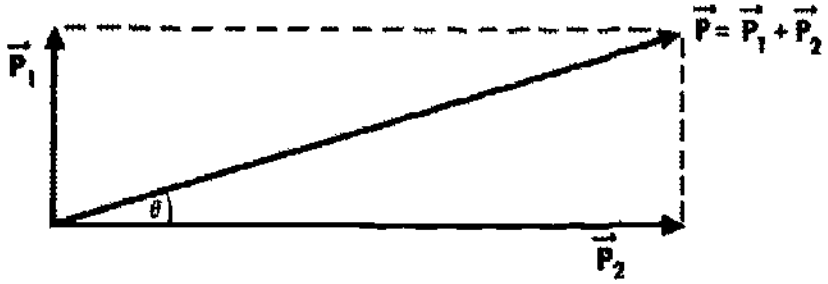
$p = \sqrt{p_1{}^2 + p_2{}^2} = \sqrt{80^2 + 60^2} = 100$ kg-m/sec

2. A 2-kg particle moves due north at a speed of 1 m/sec. A second particle of mass 10 kg moves due east at a speed of 2 m/sec. What is the direction of the total momentum of the system?

A. 18° north of east
B. 12° north of east
C. 8° north of east
D. 6° north of east

SOLUTION

Introduce the notation $\vec{P}_1$ and $\vec{P}_2$ for the momenta of the 2-kg and 10-kg particles, respectively. We want to calculate the angle θ as shown in the diagram below.



From the geometry, we have

$$\tan\theta = P_1/P_2$$

substitute values of P_1 and P_2 to find

$$\tan\theta = (2\text{ kg} \times 1\text{ m/sec})/(10\text{ kg} \times 2\text{ m/sec})$$

$$= 0.1$$

Therefore,

$$\theta = \tan^{-1}0.1 = 6°$$

3. A system of particles with masses of 8 kg and 12 kg has a total momentum of 100 kg-m/sec at 23° north of east. Determine the velocity of the center of mass of the system.

A. 100 m/sec; at 23° north of east
B. 140 m/sec; at due north
C. 20 m/sec; at 53° north of east
D. 5 m/sec; at 23° north of east

SOLUTION

The total momentum $\vec{p}$ of a system of particles is equal to the product of the total mass M of the system and the velocity of the center of mass,

$$\vec{p} = M\vec{v}_{cm}$$

Solving for $\vec{v}_{cm}$, we obtain

$$\vec{v}_{cm} = \frac{\vec{p}}{M} = \frac{100 \text{ kg-m/sec}}{(8 + 12) \text{ kg}} (23° \text{ N of E}) = 5 \text{ m/sec } (23° \text{ N of E})$$

COMPETENCE CHECK 10-2

Two particles of mass $m_1 = 2$ kg and $m_2 = 3$ kg are moving with velocities of 10 m/sec due east and 20 m/sec due west, respectively. Determine the velocity of the center of mass, $\vec{v}_{cm}$, of the system.

A. 8 m/sec; due west
B. 16 m/sec; due east
C. 16 m/sec; due west
D. 8 m/sec; due north

10-3 The Second Law in Terms of Momentum

Mass must be constant if the second law in the form

$$\vec{F} = m\vec{a}$$

is to be valid. In a number of cases, the mass of the system continually varies so that this equation can no longer be applied. For example, as a chemically propelled rocket moves, it burns fuel continuously so that its mass decreases with time. Such problems are most easily handled by applying momentum considerations and, for this reason, it is important to be able to apply the second law in momentum terms.

Newton's expression of the second law in Latin, when translated freely into modern terminology, reads

The rate at which the momentum of a body changes is proportional to the resultant force acting on the body and takes place in the direction of the straight line in which the force acts.

In equation form,

$$\vec{F} = d\vec{p}/dt \qquad (10\text{-}4)$$

If the mass is constant, the more familiar form of the second law is valid since

$$\vec{F} = d\vec{p}/dt = d(m\vec{v})/dt = m\,d\vec{v}/dt = m\vec{a}$$

Force and the rate of change of momentum must be equated to solve the following problem.

PROBLEM

The total mass of a system is 3 kg and the magnitude of the system's momentum is changing at the rate of 15 kg-m/sec^2. What is the magnitude of the net external force exerted on the system?

SOLUTION

Newton's second law of motion can be expressed as

$$\vec{F} = \frac{d\vec{p}}{dt}$$

This shows that the force exerted on a body is equal to the time rate of change of its momentum. Hence, the magnitude of the force exerted on the given system is 15 kg-m/sec^2 = 15 nt.

ENABLING PROBLEM

The total mass of a system is 100 gm, and the magnitude of the system's momentum is changing at the rate of 1000 gm-cm/sec^2. The magnitude of the acceleration of the center of mass of the system is

A. 1000 cm/sec^2
B. 10 cm/sec^2
C. 100,000 cm/sec^2
D. 98,000 cm/sec^2

SOLUTION

For a system whose mass is constant we have

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} = m\vec{a}$$

so

$$a = \frac{1}{m}\frac{dp}{dt} = \frac{1000\ \text{gm-cm/sec}^2}{100\ \text{gm}} = 10\ \text{cm/sec}^2$$

COMPETENCE CHECK 10-3

The total mass of a system is 15 kg and the magnitude of the acceleration of its center of mass is 10 m/sec^2. What is the rate of change of the system's momentum?

10-4 Conservation of Momentum

Consider a system of n particles with total momentum $\vec{P}$,

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \ldots + \vec{p}_n$$

This equation can be differentiated with respect to time to obtain

$$\frac{d\vec{P}}{dt} = \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} + \ldots + \frac{d\vec{p}_n}{dt}$$

Now introduce Newton's second law in momentum terms,

$$\vec{F}_i = d\vec{p}_i/dt$$

in which $\vec{F}_i$ is the force acting on particle i, with the result

$$\frac{d\vec{P}}{dt} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n \qquad (10\text{-}5)$$

The right hand side of this expression is the total force acting on the system. Forces between the particles due to their mutual interactions (called internal forces) may be ignored in this sum because, by the third law, the action and reaction forces are equal in magnitude but opposite in sign and their sum is zero. It follows that the right side of Eq. (10-5) is just the resultant $\vec{F}^e$ of all external forces due to agents outside of the system of particles:

$$\frac{d\vec{P}}{dt} = \vec{F}^e \qquad (10\text{-}6)$$

When the net external force is zero, we have

$$d\vec{P}/dt = 0$$

and the momentum of the system is conserved,

$$\vec{P} = \text{constant}$$

Another way to express this principle is to state that

<u>When the net external force acting on a system is zero, the total initial momentum is equal to the total final momentum</u>

or, when $\vec{F}^e = 0$, then

$$\vec{P}_{initial} = \vec{P}_{final} \qquad (10\text{-}7)$$

Note that this is a vector equation which can be reduced to components when necessary.

Problems in this set entail recognition of the fact that when the external force is zero in a given direction, then the component of momentum in that direction is conserved. The principle of momentum conservation is applied to find the final velocity of a body moving in one dimension.

PROBLEM

An 8-ton, open-top freight car is coasting at a speed of 5 ft/sec along a frictionless horizontal track. It suddenly begins to rain hard, the raindrops falling vertically with respect to ground. Assuming the car to be deep enough, so that the water does not spatter over the top of the car, what is the speed of the car after it has collected 4.5 tons of water?

SOLUTION

There are no external forces in the horizontal direction acting on the car-water system. Therefore, momentum is conserved. Thus,

$$m_i v_i = m_f v_f$$

and

$$v_f = \frac{m_i}{m_f} v_i = \frac{m_i g}{m_f g} v_i = \frac{8 \text{ tons}}{12.5 \text{ tons}} \times 5 \text{ ft/sec} = 3.2 \text{ ft/sec}$$

Note that, since the mass (or weight) of the system is involved in a ratio, no conversion to slugs (lb) is necessary.

ENABLING PROBLEM

A swimmer dives from the stern of a stationary rowboat. His mass is 70 kg and that of the rowboat 140 kg. The horizontal component of his velocity when his feet leave the boat is 3 m/sec relative to the water. What is the speed of the boat immediately after the dive?

SOLUTION

The momentum of the system is zero before the dive. In the absence of an external force, momentum is conserved during the dive; therefore, the momentum of the system after the dive is also zero. We simply have to solve the equation $m_1v_{1x} + m_2v_{2x} = 0$ for v_{2x}. Thus,

$$v_{2x} = -\frac{m_1v_{1x}}{m_2} = -\frac{70\text{ kg} \times 3\text{ m/sec}}{140\text{ kg}} = -1.5\text{ m/sec}$$

the minus sign indicating that $\vec{v}_{2x}$ is directed oppositely to $\vec{v}_{1x}$.

COMPETENCE CHECK 10-4

A block of wood of mass $M = 0.8$ kg is suspended by a cord of negligible mass. A bullet of mass $m = 4$ gm is fired horizontally at the block with a muzzle velocity of 400 m/sec. The bullet remains embedded in the block. What is the speed with which the wood block (with bullet embedded) is set into motion?

REVIEW PROBLEMS

A1. A block moves horizontally with a velocity of 2 ft/sec. Its mass is 4 slugs. What is its momentum?

A2. An object of mass 2 kg moves to the right with a velocity of 4 m/sec; another object of mass 4 kg moves to the left with a velocity of 2 m/sec. What is the total momentum of the system?

A3. The momentum of a system is changing at the rate of 5 kg-m/sec. What is the magnitude of the net external force exerted on the system?

A4. The total mass of a system is 2 kg. The momentum of the system is changing at the rate of 6 kg-m/sec. What is the magnitude of the system's center of mass acceleration?

A5. Two objects attract each other, but are not under the influence of any other forces. Which of the following statements is true?

A. the center of mass accelerates

B. the center of mass may move at constant velocity

C. the center of mass must be stationary

D. a center of mass cannot be defined for interacting particles

A6. When a group of particles is subjected to external forces, the center of mass moves as though it were a particle subjected to the sum of all the external forces. The mass of this fictitious particle is

A. the average mass of the group of particles

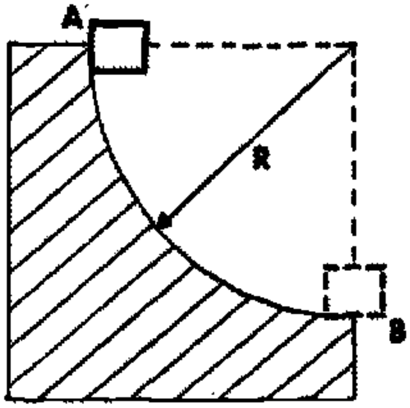
B. the mass of the heaviest particle in the group

C. the mass of the lightest particle in the group

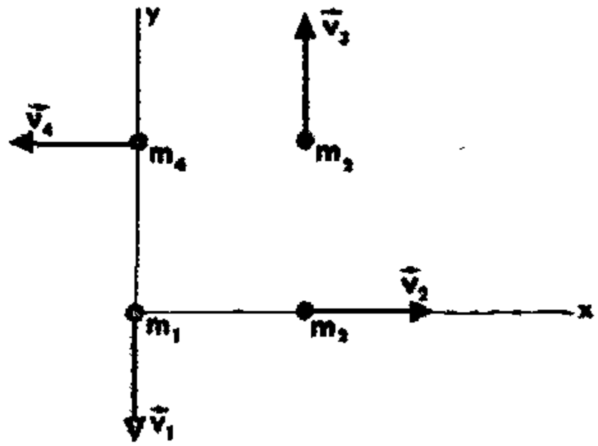
D. the sum of the masses of the particles in the group

REVIEW PROBLEMS

B1 A body with a mass of 3 kg slides down a curved track which is one quadrant of a circle of radius 1 m. If the track is frictionless and the block starts from rest, what is the momentum of the block at the bottom of the track.



B2 Four particles, each of mass 3 kg, occupy the four corners of a 4 m × 4 m square as shown in the diagram. Each particle is moving with a speed of 10 m/sec in the direction shown in the diagram.



(a) Locate the coordinates of the center of mass of the four-particle system in the coordinate system shown in the diagram.

(b) Calculate the velocity of the center of mass of this system.

B3 A force of constant direction given by

$$F = \frac{1}{2} kt^2 \text{ nt} \qquad (k = 8 \text{ nt/sec}^2)$$

is exerted on a 2-kg particle which is initially moving at a speed of 10 m/sec. Find the momentum of the particle at the end of 3 seconds.

B4 A nucleus, originally at rest, decays radioactively by emitting an electron of momentum 9.22×10^{-16} gm-cm/sec, and at right angles to the direction of the electron a neutrino with momentum 5.33×10^{-16} gm-cm/sec. What is the magnitude of the momentum of the residual nucleus?

CHAPTER 10

ANSWERS TO COMPETENCE CHECKS

10-1 8.9 kg-m/sec

10-2 A

10-3 150 kg-m/sec^2

10-4 1.99 m/sec

CHAPTER 10

ANSWERS TO REVIEW PROBLEMS

A1 8 slug-ft/sec

A2 zero

A3 5 nt

A4 3 m/sec^2

A5 B

A6 D

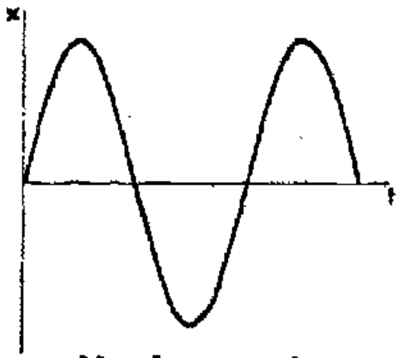
B1 13.3 kg-m/sec

B2 (a) $x_{cm} = 2m$; $y_{cm} = 2m$ (b) zero

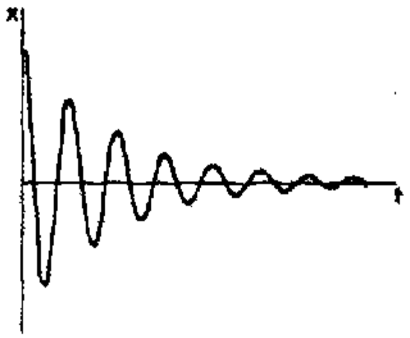
B3 56 kg-m/sec

B4 1.06×10^{-15} gm-cm/sec

frictionless systems, so that the motion or vibration can continue for an infinite duration. Such motion is termed undamped. In any real case, however, friction is present, and so the motion decreases and eventually vanishes. Such motion is termed damped. This difference is illustrated in the figures below.

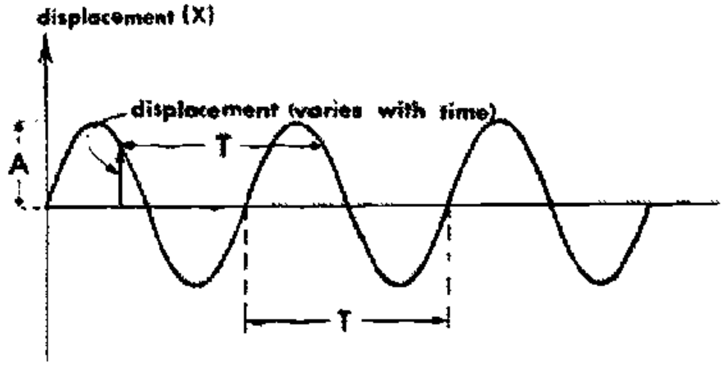


Undamped



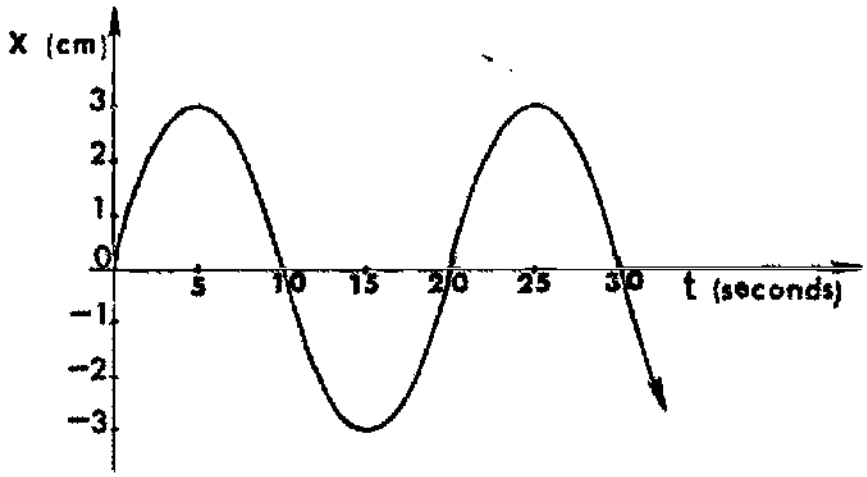
Damped

There are other quantities associated with simple harmonic motion. The first is displacement, the actual displacement of the particle from the origin. Displacement varies with time, and may be positive or negative. The second is the amplitude (A), which is the magnitude of the maximum displacement. Amplitude always has a positive value. The quantities in SHM are most easily visualized when displacement versus time is plotted, as shown in the following diagram.



The following problems are exercises in the definitions of amplitude, displacement, frequency and period, and in the relation between frequency and Period.

PROBLEM



The graph describes the motion of a particle in that it shows the displacement x as a function of time. From the graph, what is the amplitude, frequency, and displacement at t = 15 seconds?

SOLUTION

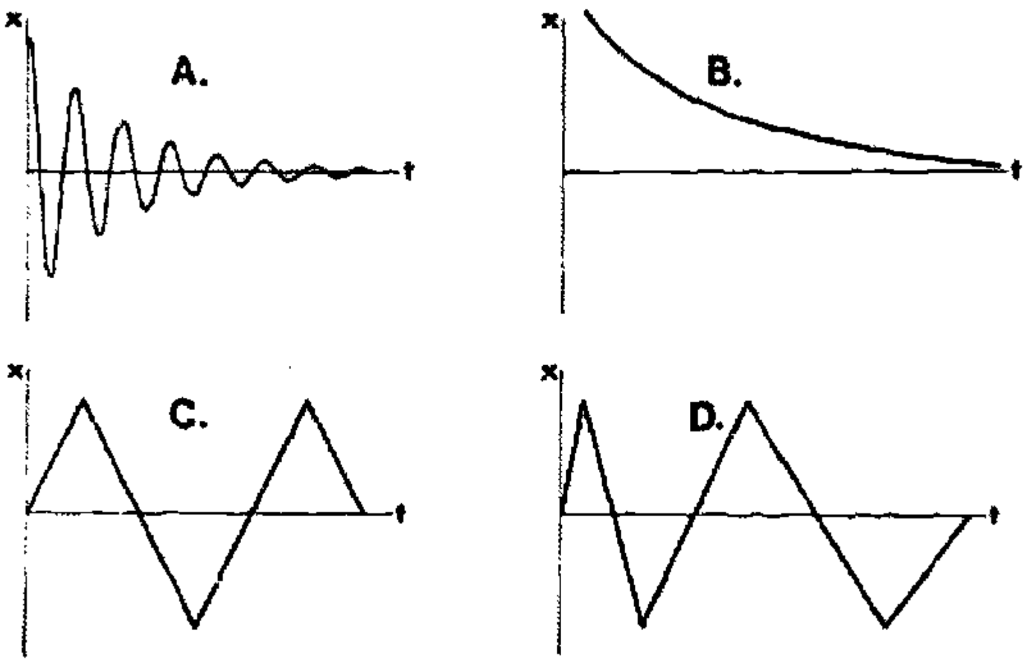
The graph shows a motion which is periodic; i.e., one which repeats itself at regular intervals. In addition it can be described by a simple trigonometric function--the sine function--and thus the motion is simple harmonic. The maximum excursion from equilibrium is 3 cm, and thus the amplitude is 3 cm. The motion repeats itself every 20 seconds; the period, or periodic time, T, is 20 seconds. The reciprocal of the period is the frequency ν, and in this case

$$\nu = 1/T = 1/20 \text{ sec}^{-1} = .05 \text{ Hz}$$

The displacement is found from the graph at $t = 15$ seconds and equals -3 cm.

ENABLING PROBLEMS

1. **The following graphs show the displacement of a particle as a function of time. Which one is executing periodic motion?**



SOLUTION

The correct answer is C.

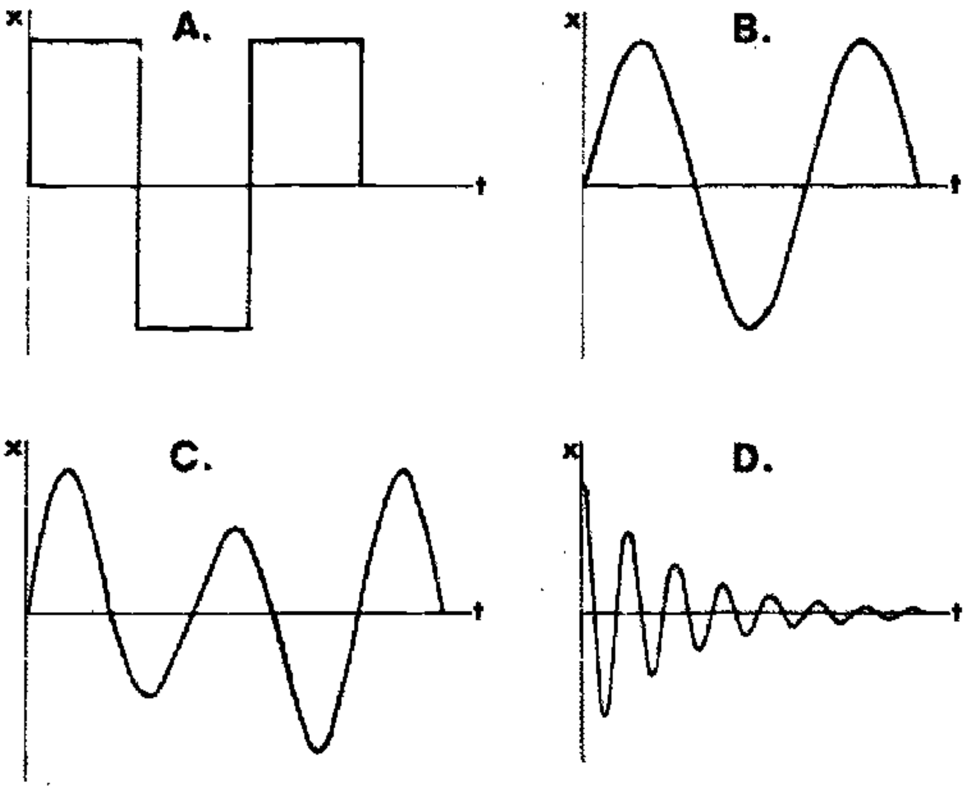
The motion repeats itself at regular intervals and so is periodic

A - INCORRECT - Notice that the vibration is decaying--no peak is as high as a previous one, and thus the motion never repeats itself. It therefore cannot be periodic in the strict sense of our definition. Such motion is often qualified as damped periodic motion.

B - INCORRECT - The motion is a slow asymptotic decay to zero. It does not repeat and therefore cannot be periodic.

D - INCORRECT - Notice the time intervals between the instances where the line crosses the x axis. They are constantly increasing, and thus the motion does not have regularity. It cannot be periodic.

2. The following graphs show the motion of a particle as a function of time. Which one describes a particle executing simple harmonic motion?



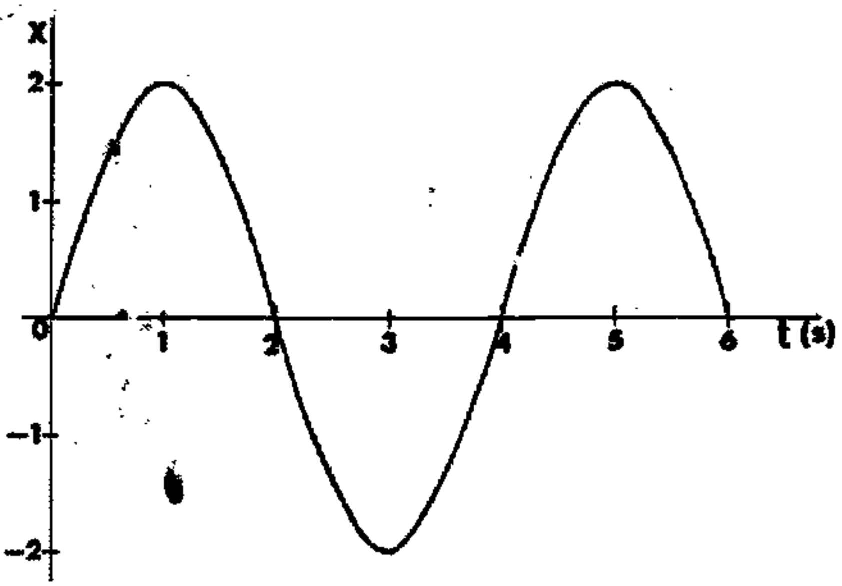
SOLUTION

B is a sine curve; since SHM is describable in terms of either a single sine or cosine function, the motion is simple harmonic.

A, C - INCORRECT - In order that the motion be simple harmonic motion, it should be expressible in terms of a single sine or cosine term. This is obviously not the case, and so the motion cannot be simple harmonic.

D - INCORRECT - This curve is a decreasing or decaying or a damped sine curve. The motion never truly repeats, and so technically the motion is not periodic. It is termed damped simple harmonic motion.

COMPETENCE CHECK - (50-1)



For the above graph showing simple harmonic motion, what is the period?

What are the frequency, amplitude, and displacement at t = 3 seconds?

50-2 The Equation of Motion of a Simple Harmonic Oscillator

In Figure 1 a mass is attached to a light (massless) elastic spring; that is, one that exerts a restoring force which is proportional to the displacement. Such a system executes simple harmonic motion, as is indicated in the diagram.

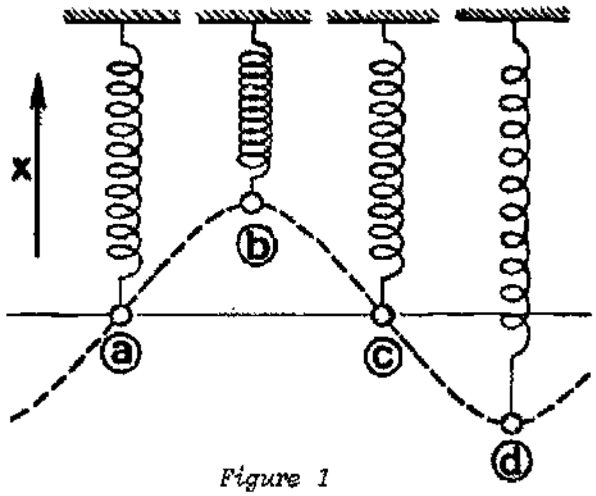


Figure 1

It is possible to describe this motion analytically by the use of Newton's second law of motion and by Hooke's law, which relates the deformation of the spring to the force of restoration exerted by the spring.

The strategy is very straight forward. Write Newton's second law for the mass, and realize that the force term in the second law is supplied by the spring. This equation is

$$m \frac{d^2x}{dt^2} = -kx$$

or, upon rearranging,

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

where

$$\omega^2 = k/m$$

The quantity ω is called the <u>angular frequency</u> of the motion for reasons which will be made apparent in the problems.

One may wonder why the force due to gravity, $-mg$, is not included in the force term since we have considered a spring which is suspended vertically. The reason is that x is a displacement of the mass from an equilibrium position which already includes the extension due to gravity. To see this, we may call the total displacement y and write the second law:

$$m \frac{d^2y}{dt^2} = -ky - mg$$

If we make a simple substitution

$$y = x - mg/k$$

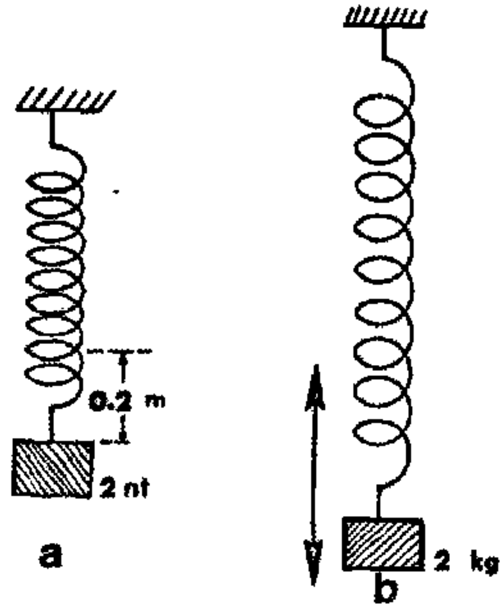
the differential equation becomes

$$m \frac{d^2x}{dt^2} = -kx$$

which is our original equation. The difference, therefore, between total displacement y and displacement from equilibrium x is a constant.

The problems which follow pertain to the derivation and solution of the equation of motion for a simple harmonic oscillator.

PROBLEM



In the diagram, a weight of 2 newtons is attached to a spring, and the extension is observed to be 0.2 meters. Next the weight is replaced by a 2 kg mass and the mass-spring combination is caused to vibrate. Find the angular frequency of the vibration and write the equation of motion for this system.

SOLUTION

The spring constant k is determined by equating the force, kx, due to the spring's elongation to the weight;

$$k(.2\ \text{m}) = 2\ \text{nt}$$

$$k = 10\ \text{nt/m}$$

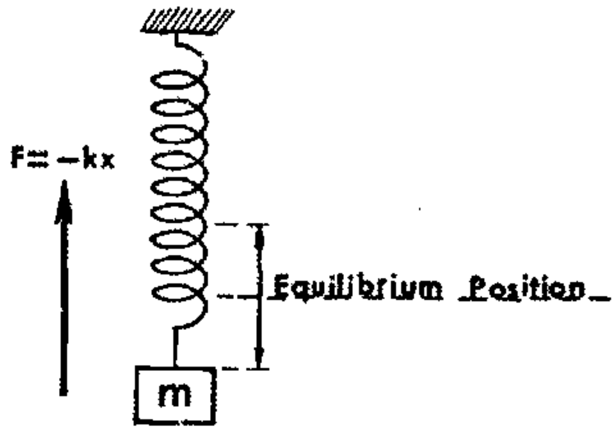
Now the angular frequency ω is found for this spring with an attached 2 kg mass,

$$\omega = \sqrt{k/m} = \sqrt{5}\ \text{sec}^{-1}$$

Newton's second law, $F = ma$, as applied to the oscillator is

$$m\frac{d^2x}{dt^2} = -kx$$

where x is the displacement from equilibrium; the restoring force is $-kx$, the minus sign implying that the force supplied by the spring is always opposite to the displacement.



The statement of the law may be written as

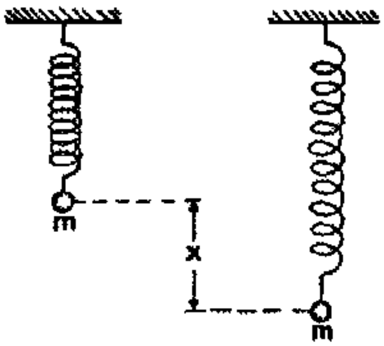
$$\frac{d^2x}{dt^2} + \omega^2x = 0$$

where $\omega = \sqrt{k/m}$. In the present case, this is

$$\frac{d^2x}{dt^2} + 5\ x = 0$$

ENABLING PROBLEM

1. In the oscillator shown, the spring has a spring constant k.



When x is the displacement of the spring, the restoring force as determined by Hooke's law is

A. kx

B. kx^2

C. $-kx$

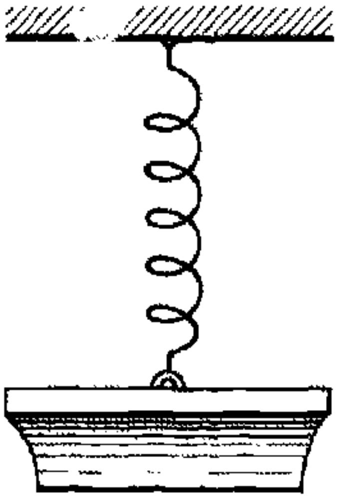
D. $1/2\ kx^2$

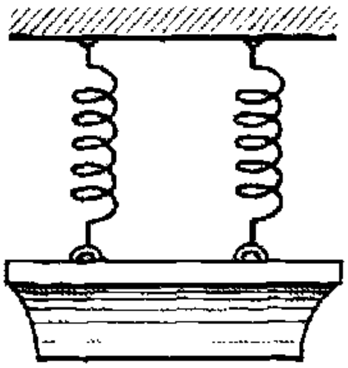
SOLUTION

The restoring force is directed opposite to the displacement and is proportional to it. Thus the restoring force must be equal to the negative (implying restoring) of a constant multiplied by displacement; i.e., $F = -kx$.

COMPETENCE CHECK - (50-2)

A weight is attached to the end of a vertically suspended spring. The extension caused by the weight is 1.22 meters. The same weight is then suspended by two springs identical to the first, as shown in the diagram. Find the angular frequency of vibration for the double-spring system and write the equation of motion.





PROBLEM

A simple harmonic oscillator has a vibrating mass m and the spring constant k. Write a general solution to the equation of motion and show that angular frequency ω and period T are related by

$$\omega T = 2\pi$$

SOLUTION

The equation of motion for the oscillator is

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

The equation is second order, homogeneous and with constant coefficients. The solution to the equation must have two arbitrary constants, and it is known that the solution is

$$x = A \sin(\omega T + \delta)$$

where A and δ are arbitrary constants.

You may verify that this is indeed a solution by differentiating twice with respect to time, and substituting in the equation of motion.

The solution may be transformed to another form by using the trigonometric identity

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

Let $\alpha = \omega t$ and $\beta = \delta$, and so obtain

$$A \sin(\omega t + \delta) = A \sin\omega t \cos\delta + A \cos\omega t \sin\delta \qquad (2)$$

Now define constants B and C so that

$$B = A \cos\delta$$

and

$$C = A \sin\delta$$

Equation (2) then becomes

$$A \sin(\omega t + \delta) = B \sin\omega t + C \sin\omega t$$

Notice that the righthand side of this result is equal to x as shown by Equation (1):

$$x = B \sin\omega t + C \cos\omega t \qquad (3)$$

Equations (1) and (3) are equivalent forms for the general solution of the harmonic oscillator equation. The motion is simple harmonic because it can be described by a single sine function as seen in Equation (1). The relation between ω and T can be found from the fact that the value of the sine function is unchanged by advancing the angle by 2π,

$$\sin\theta = \sin(\theta + 2\pi)$$

When time t advances by one period T, the value of x must be unchanged:

$$x = \sin(\omega t + \delta) = \sin(\omega[t + T] + \delta)$$

The argument of the rightmost sine function is larger than the other argument by ωT. In order for both sine functions to be equal, ωT must equal 2π:

$$\omega T = 2\pi$$

This important relation is formally identical to the defining equation for angular frequency in rotational motion.

COMPETENCE CHECK - (50-3)

A simple harmonic oscillator has angular frequency Ω. Write the general solution to the equation of motion in two forms; as a single sine function, and as the sum of sines and cosine functions. Find a relation between Ω and the period of the motion T.

PROBLEM

Two oscillators are constructed using equal masses m and identical springs of constant k. Next, one of the springs is cut in half and so the value of k is altered. Call the oscillator with the long spring A and the one with the short spring B. How are their frequencies related?

SOLUTION

From $\omega T = 2\pi$, we obtain

$$T = \frac{2\pi}{\omega}$$

therefore

$$\nu = \frac{1}{T} = \frac{\omega}{2\pi}$$

But

$$\omega = \sqrt{\frac{k}{m}}$$

so

$$\nu_A = \frac{1}{2\pi}\sqrt{\frac{k_A}{m}} \quad \text{and} \quad \nu_B = \frac{1}{2\pi}\sqrt{\frac{k_B}{m}}$$

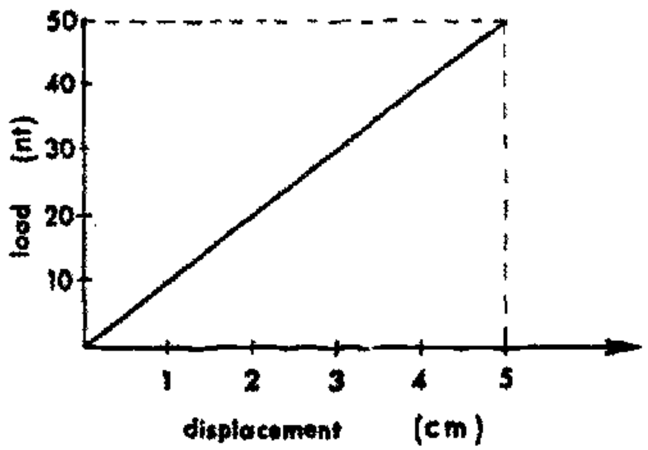
Now, when a spring is cut in half, the spring constant of each half is double that of the whole spring. (It takes twice as much force to cause unit extension.)

Using the relation $k_B = 2k_A$, we obtain

$$\nu_B = \sqrt{2}\,\nu_A$$

ENABLING PROBLEMS

1. The graph shows the elongation of a spring versus the load placed on it. The spring constant k is



A. 10^3 nt/m

B. -10^3 nt/m

C. 10 nt/m

D. 6.25×10^{-2} nt-m^2

SOLUTION

The spring constant k is the load per unit extension, or the proportionality between load and extension. Thus

$$k = \frac{50\ \text{nt}}{5\ \text{cm}} = \frac{50\ \text{nt}}{5 \times 10^{-2}\ \text{m}} = 10^3\ \frac{\text{nt}}{\text{m}}$$

2. A 10-kg mass is attached to a spring of spring constant $k = 10^3$ nt/m. What is the period of the oscillating system?

SOLUTION

The period T is determined by

$$\omega T = 2\pi$$

$$T = \frac{2\pi}{\omega} = \frac{6.28}{\omega}$$

Now

$$\omega^2 = \frac{k}{m} = \sqrt{100}$$

$$T = \frac{6.28}{10} = 0.628 \text{ sec}$$

3. A spring of constant k is cut in half. The spring constant of each half is

A. k

B. 2k

C. k/2

D. 4k

SOLUTION

Imagine the spring to be stretched by a force so that the extension is 1 m. This force is equal to k. Each half is stretched with an extension of 1/2 meter. A force of double this value is needed to stretch each half one meter. Therefore a half-spring has double the spring constant of a whole one.

COMPETENCE CHECK - 50-4

Two oscillators are constructed using equal masses m and identical springs, each of spring constant k. Next, one of the springs is shortened by cutting it and discarding 3/4 of its length. If the oscillator with the long spring is called A, and the other is B, the frequencies are related by

A. $\nu_A = 2\nu_B$

B. $2\nu_A = \nu_B$

C. $\nu_A = 4\nu_B$

D. $4\nu_A = \nu_B$

50-3 The Amplitude and the Phase Angle

The general solution to the simple harmonic equation of motion is

$$x = A \sin (\omega t + \delta) \qquad (1)$$

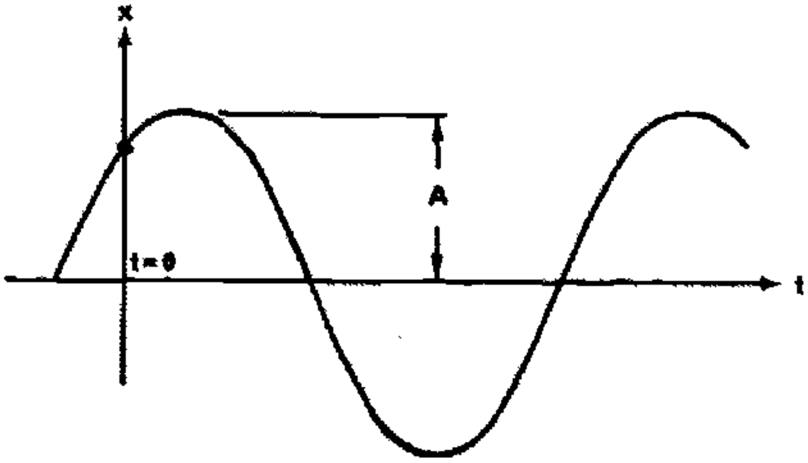
In order for this expression to apply to a specific oscillator, numerical values must be assigned to the parameters ω, A, and δ. We will see that angular frequency ω has a definite value for a given oscillator, but the constants A and δ are different for different initial conditions of the same oscillator.

In the diagram, displacement x is plotted on the ordinate axis, and the time is plotted on the abscissa. The angular frequency ω is determined by the physical constants of the oscillator; in particular, should the oscillator comprise a mass m and a spring with constant k, then ω is uniquely fixed as

$$\omega = \sqrt{k/m}$$

for that oscillator.

Amplitude A is defined as the maximum displacement of the oscillator, and is always a positive quantity. The amplitude is not fixed for a given oscillator, but is at the disposal of the experimenter who may impart a large or small amplitude at will. Amplitude is depicted in the diagram as the maximum height of the sine curve.



The constant δ is variously termed the phase constant, phase angle, or simply phase. Like amplitude, phase δ is another arbitrary constant of integration (a second order differential equation must have two) and merely describes the state of the motion at the instant the experimenter started the time-clock ($t = 0$). δ too is at the experimenter's disposal, and it too does not affect the frequency.

The problems below require a knowledge of the basic displacement equation (1) in order to recognize or calculate amplitude, phase, and frequency.

PROBLEM

A simple harmonic oscillator is released from rest at a displacement of 3 cm. The angular frequency of the oscillator is $\pi/4$ radians/sec. What is the displacement of the oscillator after 2 seconds?

SOLUTION

An expression for the displacement of a single harmonic oscillator is:

$$x = A \sin(\omega t + \delta) \qquad (1)$$

In order to find the amplitude A and phase angle δ, we express the initial conditions (the conditions at $t = 0$) in equation form:

initial displacement $\quad 3 = A \sin\delta \qquad (2)$

initial velocity $\quad 0 = A\omega \cos\delta \qquad (3)$

The last expression for velocity is obtained by differentiating x with respect to time,

$$v = dx/dt = A\omega \cos(\omega t + \delta)$$

and then setting time to zero.

Equations (2) and (3) can be solved for two unknowns, A and δ. From equation (2) we see A cannot be zero;

$$0 = \cos\delta$$

This is satisfied by

$$\delta = \pi/2 \qquad (4)$$

Putting this value into Eq. (1), we have

$$3 = A \sin\pi/2$$

or

$$3 = A \qquad (5)$$

Putting the values A, δ, and ω into Eq. (1) gives an expression for the displacement of the oscillator (in centimeters) at any time in its history,

$$x = 3 \sin(\pi/4\ t + \pi/2)$$

For the case at hand,

$$t = 2 \text{ sec}$$

and we obtain

$$x \text{ (at two seconds)} = 3 \sin\pi$$
$$= -3 \text{ cm}$$

ENABLING PROBLEM

Find an expression for the velocity of a simple harmonic oscillator which is governed by the displacement relation

$$x = A \sin(\omega t + \delta)$$

SOLUTION

This is an exercise in differentiation. Recall the simple formula

$$\frac{d}{dt}(\sin\mu) = \cos\mu\left(\frac{d\mu}{dt}\right)$$

In this case,

$$\mu = \omega t + \delta$$

and

$$\frac{d\mu}{dt} = \omega$$

Finally,

$$\begin{aligned} v &= dx/dt \\ &= A\frac{d}{dt}\sin\mu \\ &= A\cos\mu\left(\frac{d\mu}{dt}\right) \\ &= A\omega\cos(\omega t + \delta) \end{aligned}$$

COMPETENCE CHECK - (50-5)

A simple harmonic oscillator has an angular frequency of $\pi/4$ radians/sec. Initially, it is at the equilibrium position ($x = 0$) and moving with velocity π ft/sec. Find the position of this oscillator after two seconds have elapsed.

CHAPTER 50

REVIEW PROBLEMS

A1. Define periodic motion.

A2. Define frequency of oscillation.

A3. Define simple harmonic motion.

A4. Define amplitude of simple harmonic motion.

A5. The equilibrium position of any oscillator is

A. the maximum displacement

B. the point at which net force is zero

C. the position at which speed is least

D. always the central point of the motion

A6. What kind of force causes a particle to execute simple harmonic motion?

CHAPTER 50

REVIEW PROBLEMS

B1 A particle executing simple harmonic motion vibrates with a period of 1 sec and the maximum distance between any two points of the particle's path is 4 cm. Find the amplitude and angular frequency of the motion.

B2 A .5 kg mass oscillates while suspended from a spring with a spring constant of 2 nt/m. What is the angular frequency of oscillation?

B3 Write a general expression for the displacement of a simple harmonic oscillator and identify the symbols used.

B4 An oscillator with period 2 sec is constructed from a 1 meter spring. What length of spring should be discarded to reduce the period to 1 sec?

B5 Find the velocity at $t = 3$ sec of a simple harmonic oscillator governed by the displacement expression (in meters)

$$x = 5 \sin(2t + .28)$$

CHAPTER 50

ANSWERS TO REVIEW PROBLEMS

A5 B

A6 A restoring force which is proportional to the displacement of the particle from the equilibrium position.

B1 2 cm, 2π radians/sec

B2 2 radians/sec

B3 $x = A \sin(\omega t + \delta)$

x is displacement

A is amplitude

ω is angular frequency

t is time elapsed

δ is the phase constant

B4 .75 m

B5 10 m/sec

CHAPTER 50

ANSWERS TO COMPETENCE CHECKS

50-1 4 sec, .25 Hz, 2 cm, -2 cm

50-2 4 radians/sec, $d^2x/dt^2 + 16x = 0$

50-3 $x = B \sin\Omega t + C \cos\Omega t$, $x = A \sin(\Omega t + \delta)$, where A, B, C, δ are arbitrary constants

50-4 B

50-5 π ft